Calc 3
Spring 2004
Review Sheet for Exam 3

The final exam will be constructed as follows. About 2/3 of the exam will cover just the material from sections 13.1 - 13.7 and 14.1 - 14.4. The first three sections of 13 have already been covered in the previous exam, and so will receive less emphasis than the more recent material, but I will feel free to ask questions from all parts of chapter 13. This part of the final exam will be similar in intent and structure to the previous two exams. The remaining 1/3 of the final exam will cover material from the course as a whole. This will stress the important ideas and concepts from the entire course, including concepts of vectors and vector operations, parametric curves and their properties, partial derivatives, two and three dimensional geometry of curves and surfaces, plus limits, continuity, and differentiability.

This review sheet will outline the main ideas and techniques from chapters 13 and 14. However, it is not intended as a review for the entire course, and does not outline the material that will be covered in the cumulative 1/3 of the final exam.

Multiple integration

1. Conceptual meaning of double and triple integration of a function f over a region R as a limit of sums. The terms of the sum correspond to small subsets of the region (rectangles in 2D and boxes in 3D), and each such term is the product of the area or volume of the subset multiplied by a representative value of f for some point within the subset. The limit is formed as the size of the subsets goes to 0. For a region in the plane we can interpret the double integral as the volume under the graph (which exists in three space); for a triple integral there is no analogous interpretation. Integrals can also be understood as a kind of reductionism: ascribe to each point of the region some quantity that we wish to agregate over the region. The contribution from a single point is infinitely small, and is generally computed in terms of an infinitely small area dA or volume dV. The integral allows us to add up these infinitely many infinitely small quantities and determine the total for the region as a whole.
2. Setting up, interpreting, and reordering limits of integration
3. Computing double and triple integrals
4. Using polar, cylindrical, and spherical coordinates
5. Computing moments, mass, center of mass using the concept that the mass concentrated at a single point is expressed in the form $dm = \delta dA$ or $dm = \delta dV$ where δ is a density function (e.g. in grams per square or cubic inch) and dA and dV are single point amounts of area or volume, respectively.
6. The general formula for changing variables in a double or triple integral using the Jacobian determinant.

Representative Problems

1. Review any quizzes, sample problems, and extra assigned problems from class or the webpage; especially the problems connected with making a change of variables in a multiple integral using the Jacobian of the change of variables transformation
2. Set up appropriate triple integrals to determine the centroid of the region that lies inside the cylinder $x^2 + y^2 = 4$, above the xy plane, and below the sphere of radius 4, centered at (0,0,8)
3. Problems 41 page 1057, 54 page 1058
4. Problem 12 page 1054

Line integrals and vector fields

1. Concept of integrating a scalar function along a curve in the domain of the function, either in the plane or in space. As for double and triple integrals, this can be understood as a limit of approximating sums, where each term is a small increment of distance along the curve multiplied by the function value at a point of the curve.
2. Integration formula: $\int_C f dS = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|\mathrm{dt}$
3. Concept of independence of parameterization: two different parameterizations of the same curve C produce the same value for $\int_C f dS$
4. Mass and moment computations with $dM = \delta dS$
5. Concept of a vector field, and interpretation as a flow or a force field.
6. Concepts of $\int_C \mathbf{F} \cdot \mathbf{dr}$ and $\int_C \mathbf{F} \cdot \mathbf{dn}$; computation formulas for these; interpretation as circulation and flux
7. Path Independence, Conservative fields; line integral $\int_C \mathbf{F} \cdot \mathbf{r}$ when $\mathbf{F} = \nabla f$ is the difference in the values of f at the endpoints of the curve C; mixed partial tests for a field to be conservative; construction of the potential function f, when it exists
8. Concepts of flux density = divergence, and circulation density = curl; formulas divergence $= \nabla \cdot \mathbf{F} = M_x + N_y$ and curl $= \nabla \times \mathbf{F} = N_x - M_y(\mathbf{k})$ for $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$
9. Greens theorem in flux and circulation forms

10. Using Green's theorem to convert a line integral into a double integral, or vice versa

Representative problems

1. A wire is bent in the shape of the curve $y = x^2$ between the points (0,0) and (1,1). Assuming the wire has constant density of 5 gm per cm, find the center of mass.

2. A force is given in space by the equation $\mathbf{F}(x, y, z) = (5 - x)\mathbf{i} - (6 + y)\mathbf{j} + (7 - z)\mathbf{k}$. Find the work done by the force over the helical path $\mathbf{r}(t) = \cos(2\pi t)\mathbf{i} + \sin(2\pi t)\mathbf{j} + t\mathbf{k}$ as t goes from 0 to 1.

3. A fluid flow is defined by the equation $\mathbf{F}(x, y) = -\dfrac{y}{\sqrt{x^2 + y^2}}\mathbf{i} + \dfrac{x}{\sqrt{x^2 + y^2}}\mathbf{j}$. Compute the flow around and the flux across the triangular path going from (1,1) to (4,1) to (1,6) and back to (1,1)

4. Problems from page 1083: 3,10,13

5. Let $f(x, y, z) = sin(xy) - ze^x/y$, and define $\mathbf{F} = \nabla f$. Determine the value of the line integral $\int_C \mathbf{F} \cdot \mathbf{dr}$ along the straight line from (1,1,1) to (5,4,3).

6. Let $\mathbf{F}(x, y) = -\frac{y}{\sqrt{x^2+y^2}}\mathbf{i} + \frac{x}{\sqrt{x^2+y^2}}\mathbf{j}$. Show that $\mathbf{F}$ satisfies the test for exactness. Also compute directly the line integral of $\mathbf{F}$ once around the unit circle using the parameterization $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$ and show that the result is **NOT** 0. Given theorem 2 and the test for conservative fields, how is this possible?

7. Page 1093, problems 5, 17