On the consistency of Information-Matrix-type tests

Werner PLOBERGER

Dept. of Economics, Washington University in St. Louis

It is well known that the Fisher – information matrix of a parametric model can be expressed in two ways, namely as the limit of the averages of the second derivatives of the log-likelihood multiplied by -1, and as the average of the outer products of the first derivatives. The difference between these two representations is asymptotically normal, which was used to construct a specification test (White(1982)). This test has a lot of interesting optimality properties and was adapted, improved a lot.

In this paper, I investigate the situation where the test accepts. I assume that we have given a parameterized family of functions, which represent candidates for the density of our data. I assume that the test statistic does not increase too much with the sample size and that some mixing conditions are fulfilled.

Under these assumptions, I show that we can construct a parameterized family of probability measures which contains our data-generating process and whose likelihood functions equal our given ones up to a factor not depending on the parameters.

This result can be interpreted as some kind of consistency result. When the test statistic remains relatively small, then the model is in a certain sense correctly specified.

I then discuss some generalizations and applications of the theoretical results.