AMERICAN UNIVERSITY
Department of Economics



# COMPREHENSIVE EXAMINATION IN PRELIMINARY THEORY

Instructions: Please answer *all* questions and show *all* of your work.

1. Consumers in Utopia derive utility from consumption of the *n* products in their society according to the utility function:

$$U = [\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

   Where $x_i$ is the quantity consumed of the $i^{th}$ product.
   a. Define $p_i$ as the price of the $i^{th}$ product, and *m* as each consumer's total income level. Calculate the Marshallian (uncompensated) demand for good *i*, as well as the indirect utility accruing to the consumer.
   b. Assume that the price of good 1 increases from $p_1$ to $q_1$ due to a tax increase. How much money would the government of Utopia have to give to each consumer in order to compensate them for this price increase?

2. In the city of Metropolis, entry is restricted into the market for taxicabs, and a driver is permitted to operate a cab only if she has a medallion. The total number of medallions has been fixed at 1000 for 25 years, but medallions can be bought and sold. The current price is $100,000. The mayor is concerned about the shortage of cabs and has come up with a novel plan. Based on careful study that shows that, without entry restrictions, there would be twice as many cabs in operation, the mayor wants to double the number of medallions issues. Realizing that the current taxicab operators are unlikely to welcome increased competition, the mayor proposes that an additional medallion be issued to the new owner of each current medallion. The owner will then be free to do whatever she wants with the new medallion, such as selling it.

   You are the economic advisor to one of the 1,000 medallion owners. She wants to know whether to support the mayor's plan or oppose it. What do you tell her and why?

3. Jack lives eight miles from the Mississippi River, while Jill lives 12 miles from the Mississippi River. The two neighbors are separated by 15 miles along the

river. Pumphouses cost $P each, and must be located on the river. Laying pipe costs $100 per mile. Once the pipe is laid, and the pumphouses are installed, water is available at no extra cost.

a. Do the neighbors have an incentive to minimize the total (to both neighbors) cost of obtaining water?
b. If one pumphouse is used to supply both neighbors, show that it will be located six miles from the point on the river closest to Jack's house. What will the cost of water be for each farmer and in total in terms of $P?
c. Suppose the neighbors build their own pumps. What will the cost to each be, as well as the total cost?

4. Consider the following Keynesian model with wealth effects:

$Y = C + I + G$, where

$C = C(Y - T, W) = \frac{1}{2}(Y - T) + \alpha W$

$I = 100 - r$

$\frac{M}{P} = L(Y, r, W) = \frac{1}{2}Y - r + \beta W$

where $W = M + B + K$ denotes wealth, Y output, C consumption, I investment, r the interest rate, G government spending, M money supply, B government bonds, K capital, and P the price level. The parameters $\alpha$ and $\beta$ indicate the sensitivity of consumption and money demand to "wealth effects."

Assume the following values: $P = 1$ and $G = T = M = K = 0$. We are normalizing fiscal spending, taxes, money supply, and capital to zero.

a. Derive the IS and LM curves.
b. Solve the IS and LM model under the assumption that $B = 1$. In particular, derive equilibrium Y and r as functions of the parameters $\alpha$ and $\beta$.
c. Show your results in part b. in an IS/LM graph.
d. Suppose that $B = 2$ (that is, imagine that the government has issued more bonds to finance interest payments on existing debt.) In your graph, show how the IS and LM curves would be affected by an increase in B.
e. Under what parameter restrictions on $\alpha$ and $\beta$ would equilibrium Y actually fall as B increases? Provide an intuitive explanation behind this result.

5. Consider the following quality ladders model with a <u>fixed</u> continuum of goods:

a. At each point in time, consumer expenditures E are given by the following budget constraint:

(1) $E = \int_0^1 p_j x_j \, dj$

where $x_j$ is the quantity of the jth good and $p_j$ the corresponding price. The consumer's *intratemporal* utility function is given by:

(2) $u = \int_0^1 \log(q_j x_j) \, dj$

where $q_j$ denotes the quality level of the jth good. Maximize (2) subject to (1), by choice of $\{x\}$, to derive the demand for each good.

b. Suppose in the typical industry, the state-of-the-art good is $\lambda$ times better in quality than the next highest quality good. Suppose the marginal cost of producing a good equals the wage w (since the production function is such that it takes one unit of labor to produce one unit of a good). Why would the leading producer charge a price $p = \lambda w$?

c. Normalize E = 1. Calculate the profits of the leading firm.

d. The value of a firm is

(3) $v = \int_0^\infty e^{-(\iota+r)t} \pi_t \, dt$

where $\iota$ is the rate of innovation and r the interest rate. Explain why profits are discounted at the rate $\iota + r$, and not just r? Using equation (3), find the steady-state value of v.

e. A firm that invests $\alpha\iota$ units of labor in R&D has a probability of a successful technological breakthrough of $\iota$ and a probability of failure of $(1 - \iota)$. What is the expected value (or *benefit*) of investing in R&D and what is the *cost*? Explain why the free-entry condition is $v = w\alpha$?

f. The economy has a fixed number of workers, L, that can work either in R&D or in manufacturing goods x. The economy-wide resource constraint is therefore:

$L = L_X + L_{RD}$

Find $L_X$ and $L_{RD}$.

(g) Use your results in (c) – (f) to derive the equilibrium innovation rate, $\iota^*$. Discuss how/why the rate of innovation depends on L, $\alpha$, $\lambda$, and r. Is there a "scale effect"? Discuss the plausibility of such an effect.