Theory must transcend empiricism in order to be theory at all.

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I.

SOCIOLOGY AS A STRICT SCIENCE

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Sociology and Ontology

Sociology and philosophy seem to have this in common: in spite of notable progress in special fields, the discipline as a whole lacks unity and fails to accumulate knowledge. Both disciplines are split into separate schools between which there exist no clearly defined relations and no notable increase of insight.

Thus after an auspicious beginning Pitirim Sorokin’s (1927, 1942, 1947) attempt to create a sociological theory in the vein of Descartes’ analytical geometry by resolving (real) “multibonded” “sociocultural groups” into (abstract) “unibonded” relations failed to produce a convincing result for lack of logical rigor. Sorkin would have failed even if he had not subsequently (1937-41) attempted an “integral” theory of “ideational,” idealistic, and “sensate” “Supersystems” which looked like a parody of Comte’s law of the three stages. For similar reasons Talcott Parsons’ (1937) attempt to look for a common denominator for the classics in social theory (viz. Alfred Marshall, Pareto, Durkheim and Max Weber) and subsequently (1951, 1960, 1966, 1977) to elaborate a theory of action systems seems to have come to nothing. One is tempted to ask, “who reads Parsons nowadays?”

Sociological theory continues to veer between diverse paradigms which seem to resist any attempt to integrate them into a unified paradigm (e.g., G. Ritzer 1981; J.Alexander 1982-83; R.Collins 1984, A.Giddens 1984). With postmodernism and poststructuralism as the latest offspring, disarray and eclecticism seem to reign supreme and discourage any further attempt to elevate sociology to a rigorous science.

At the same time a closer look at philosophy may help us to solve problems which sociological theory has failed to solve or even address. From Plato and Aristotle to Husserl (1913) and Nicolai Hartmann (1949/1953), Mario Bunge (1962) and Ervin
Laszlo (1972) a general discipline has existed which purports to encompass all special disciplines: ontology—an aim which ontology shares with positivism, Marxism and systems theory, which may be considered cryptoontologies.

Unfortunately Aristotle’s ontology culminates in metaphysics and in speculation about God as the prime mover and ultimate telos, which has made it anathema to positivists and neo-Kantians alike. Half a century after Husserl, Scheler and Nicolai Hartmann no revival of ontology is in sight although structural Marxism (Sebag, Godelier and Althusser; Jon Elster, John Roemer, Erik Olin Wright, Gerald A. Cohen, Allen Wood) and Systems theory (Laszlo, Boulding, Bunge, L.L. Whyte, J.G. Miller, Buckley, Bailey; Luhmann) may be considered legitimate offspring of ontology.¹

The case of sociology as a strict science is therefore not as hopeless as it might seem.² After all, Auguste Comte was an ardent champion of “first philosophy,” as his English counterpart Herbert Spencer (1862) was an ardent champion of First Principles and as the fathers of “scientific socialism” were ardent champions of “dialectical materialism.” From Bacon to Descartes, Leibniz and Husserl “first philosophy” denotes the one encyclopedic discipline that comprises all others. As Auguste Comte argued, “positive philosophy” is the science whose specialty it is to be general.³ As such it constitutes the secularized successor of theology and metaphysics. It is in this sense that Comte as well as Marx and Engels pursue the tradition of ontology, which is marked by encyclopedic scope, eidetic (structural) reduction, and hierarchy.

**Auguste Comte’s Encyclopedic Law**

As one of its distinctive features Comte’s (1844) positivism includes his “encyclopedic law,” which underlies his idea of the hierarchy of sciences. The latter is founded in the hierarchical structure of reality such that the more complex is founded on, and thus presupposes, the more simple. In this way the hierarchy of the sciences is founded in the hierarchical structure of reality, which ascends from inorganic to organic, and finally, supraorganic (social and ethical) reality. As Comte put it in the Introduction to his *Cours de Philosophie Positive* (1975, p.93):
All observable phenomena may be included within a very few categories, so arranged as that the study of each category may be grounded on the principal laws of the preceding and serve as the basis of the next ensuing…

For the uninformed reader Comte’s encyclopedic law thus seems to result in the well-known hierarchy of the sciences, which starts with mathematics and proceeds through astronomy, physics, chemistry and physiology (biology) to “social physics,” which was subsequently supplemented by ethics. A closer look at Comte’s work, especially as he summarized it in his *Discours sur l’Esprit Positif* (1844), lends itself to a more comprehensive interpretation: The encyclopedic, or hierarchical law combines the synchronic (static) principle of dependence (i.e., foundation) with the diachronic (dynamic) principle of evolution such that each science implies not only a different methodology for each science, but a different structure of reality.

Comte’s encyclopedic law thus represents the positivist version of classical ontology, whose ascending order of matter, flora, fauna and reason implies the same principle of foundation as does Montesquieu’s *Spirit of the Laws* and Marx’ paradigm of socio-economic basis and ideological superstructure. In each case the higher, more complex superstructure (SS) depends on a less complex basis (BA) without which it could not exist. In all three instances an ontological hierarchy is based on a logical calculus which connects necessary condition and sufficient reason, i.e.

\[
\text{“B only if A”} = A \iff B = \neg A \rightarrow \neg B = B \rightarrow A.
\]

\[
= \text{ necessary condition} \iff \text{sufficient reason}.
\]

\[
= \neg \text{ necessary condition} \rightarrow \neg \text{sufficient reason}
\]

\[
= \text{ sufficient reason} \rightarrow \text{ necessary condition}.
\]

A similar relationship reverberates in Parsons’ (1966, 1972) parameters of condition and control, energy and information, and lastly, in his “Paradigm of the Human Condition” (1978), which he equated with cybernetics. Indeed, cybernetics is predicated on the foundational function, which is asymmetrical: Obviously, every cybernetic system (SYS) implies an environment (ENV) from which it arises, i.e.,

\[
\text{ENV} \iff \text{SYS} \rightarrow \text{SYS} \rightarrow \text{ENV}.
\]
Nicolai Hartmann’s New Ways of Ontology
Based on an extensive study of Hegel, Nicolai Hartmann’s (1933, 1940, 1953) “fundamental law of ontology” has put Comte’s hierarchic law and Marx’ materialistic interpretation of history on a new basis which sociological theory has failed to notice only to its own detriment.

Dismissing immutable Platonic ideas and Aristotelian essences, Hartmann discerned four main strata (Schichten) of Being, which are defined by ”specific dominant ontological categories which distinguish the strata from each other” (1953, p.47). In other words, Hartmann’s categories are the equivalent of distinctive qualities that serve as mapping functions for empirically verifiable sets. Cantor finally replaces Parmenides.

While the four strata of Hartmann’s ontology differ little from Aristotle’s theory of the psyche, he has made an ingenious contribution to analytical theory, which has unfortunately gone unnoticed. Hartmann distinguishes two different forms of foundational relations, viz. “Überbauung” and “Überformung.” In the first case a superstructure emerges from a basis which it supplements without encroaching on it. In the second case the superstructure rises to power over its basis, upon which it imposes a new form and thus relegated it to subsidiarity while it continues to depend on it.

The first case is illustrated by primitive societies of hunters and gatherers, which continue to be an integral part of Nature whatever novelties emerge. In the second case, starting with Gordon Childe’s “Neolithic revolution” of domesticating animals and growing crops, Nature is converted into a subsidiary that is “superformed” (überformed) by Society, which dominates Nature while it continues to depend on it.

In brief, Überbauung (emergent superstructure) is marked by openness, i.e., novelty and dependence. By contrast, Überformung is marked by closure, i.e., domination and “novel substrata that serve as matter” (1953, p.69), i.e., by reduction to subsidiarity. For clarity’s sake, let us replace Hartmann’s original terms by the duality of *emergent and dominant superstructures.

Accordingly, just as there exist two possible relationships between Nature and Society, there likewise exist two possible relationships between the Division of Labor (production) and Civil Society as well as between Civil Society and the State. While Marx
took it for granted that Civil Society and the State are “determined” by the “Mode of Production,” he failed to recognize the difference between dependence and dominance. Accordingly exploitation is not “determined” by the Mode of Production. Rather it implies the dominance of Civil Society over Production. In no way does the Mode of Production dominate Civil Society, nor does it dominate the State, as Louis Napoleon’s coup of the 18th Brumaire illustrated.

This observation corrects orthodox Marxist theory and its overblown concept of the “mode of production,” but it does not wholly refute it. Its dominance notwithstanding, Civil Society continues to depend on Production, which in fact “determines” Civil Society “in the last analysis,” just as the State depends on Production and Civil Society, which indeed “determine” the State “in the last analysis,” even though they are reduced to subsidiaries – a fundamental error which took a Lenin to correct it.4

The same holds for the relationship of Society and social Consciousness. Even where the latter rises to dominance and aims to shape Society it continues to depend on it. As Bukharin (1924) noted, after the October revolution the sequence of determination was reversed: Leninism now determined State Power, Civil Society, and the Mode of Production, all of which were reduced to subsidiarity.

On the other hand, before rising to dominance, consciousness has to emerge in the first place, in which case it closely reflects the interests of those on whose support it depends. Marx’ conception of social consciousness thus reflects emergent consciousness in contrast to that of Hegel -- and incidentally, that of Plato and the entire Stoa -- who wanted Justice, i.e., moral consciousness, to dominate Society, so that in terms of Rousseau it might be replaced by right.5

At the same time the rise to dominance is by no means an automatic process, as Gramsci realized when discussing ideological hegemony. While every class and faction tends to develop its own kind of consciousness, it must reach out for outside support if it wants to rise to dominance, in the pursuit of which it has to upgrade its ideology and let Justice and the common good take pride of place over class interests.6

It is for this reason that Anglo-Saxon democracy differs fundamentally from continental European democracy: In the latter, the voters choose between various parties whose ideologies reflect factional interests, while in the former, the voter chooses be-
tween two candidates each of whom must address the common interest in order to win the election. In the outcome the political and social consciousness is shaken up every four years whereas in the continental political systems the ideologies tend to endure and government is determined by changing coalitions which have little appeal to the voters. In any case we note two kinds of democracy dependent on two different kinds of hierarchy.

Much more will have to be said about the importance of the foundational relationship. Yet two crucial insights already stand out: No strict sociological theory is conceivable without the hierarchical relationships between the various domains and dimensions of society, as the relationship between Production, Civil Society and State power has shown. No less important, a fundamental duality exists between free-floating, imaginative emergent superstructures and constrained, rational dominant superstructures, a division which underlies the freedom of choice as anticipated by Pareto’s residues of combination and persistence, W. James’ duality of the “once born” and the “twice born,” and Max Weber’s famed antithesis of mysticism and asceticism.

**Factual Truth and Rational Truth**

Besides being encyclopedic and hierarchical ontology harbors a third distinctive feature which it shares with analytical theory: the reduction of empirical facts to rational truth. Widespread misconceptions to the contrary, it is not the task of science to “mirror,” reflect, etc. reality any more than Mendeleev’s Periodic Table of Elements mirrors matter. Rather, it is the function of science to reveal the structure that underlies our knowledge of the external world. Like the longitudes and latitudes of the globe, theory serves to come to grips with reality without being a part of it.

To elucidate the meaning of logical truth a short glance at Popper’s (1934) *Logic of Scientific Discovery* is illuminating. Popper went to great length to show that universal statements elude verification. According to Popper they are validated only indirectly by not being contradicted; i.e., they are valid only until further falsification. Popper thus refuted all claims of universal truths to be immutable. Yet while he was right in refuting the myth of immutable truth *a priori*, he did not go to the bottom of the problem. To say “all swans without exception are white” does indeed elude verification. What Popper failed to realize is that universal statements are not founded in the absence of empirical
falsification, but in logic, viz. in the fact that universal statements (∀) share the same truth-values with the dual denial of their existential (∃) counterpart, viz.

\[ ∀(x) \, Q(x). \; = \; ∼∃(x) \, ∼Q(x), \]

just as existential statements (∃) share the same truth-values with the dual denial of their universal converse, viz.

\[ ∃(x) \, Q(x). \; = \; ∼∀(x) \, ∼Q(x). \]

By the same token to say “there exists some swan which is not white” is tantamount to saying “not all swans are white,” which contradicts the universal statement

\[ ∃(x) \, ∼Q(x). \; = \; ∼∀(x) \, Q(x). \]

While universal and existential statements are mutually irreducible, they explain each other. What is usually termed “quantification” thus turns out to denote judgment. All our knowledge is either universal (i.e., apodictic) or existential (i.e., contingent), as the two quantors ∀ and ∃ indicate.

**Quantification and Modal Logic**

As Leibniz (1714, § 33) put the same problem in his *Monadologie*, “There exist two kinds of truth, vérités de raison and vérités de fait. The first are necessary and their contrary is impossible; the second are contingent and their contrary is possible.”¹¹ As Leibniz’ formulation elucidates, quantification is homologous with modal logic. To say “all swans are white” is tantamount to “swans are necessarily white,” and “it is impossible that there exists any swan that is not white,” i.e.,

\[ ∀ \, Q(x) : ∃ \, Q(x). \; = \; N \, Q(x) : P \, Q(x). \]

Accordingly,

\[ ∀ \, Q(x) \; = \; ∼∃ \, ∼Q(x). \; = \; N \, Q(x) \; = \; ∼P \, ∼Q(x), \]

and

\[ ∃ \, Q(x) \; = \; ∼∀ \, ∼Q(x). \; = \; P \, Q(x) \; = \; ∼N \, ∼Q(x). \]

Modal logic thus teaches us an important lesson: Necessity does not inhere in reality, but in our judgment about it. By the same token, logical truths are not so much necessary as apodictic, just as factual truths are not so much possible as contingent. To say “x is necessarily Q” is to say “it is impossible (inconceivable, unthinkable, etc.) that x is non-Q,” just as saying “x is necessarily non-Q” is to say “it is impossible that x is Q.”
Whatever the meaning of (x) or Q, the logical function that connects them is amenable to explanation regardless of the meaning of the arguments. Logical truth is therefore not founded in empirical verification, but in the truth values which define each of the sixteen truth functions. By the same token, once it is understood that necessity and possibility, apodicticity and contingency inhere in our judgment rather than in reality, the myth of immutable essences dissolves. Rather than reflecting reality, objectivity is a function of judgment that has nothing to do with quantification.

In contrast to the Mind, which is based on personal sentiments and values, the Intellect is coextensive with impersonal objectivity because it is based on the invention and use of artifacts none of which could come about without a subject that produces them. Starting with ant heaps and bee hives, dens and beaver dams, primitive signals and gestures, tools, weapons and buildings, and culminating in language, symbols and calculi, the Intellect is constitutive of what Hegel called “materialized,” “externalized” or “objectified” Mind – an activity which Lukács wrongly equated with alienation.

While language is without doubt the instrument par excellence for developing and practicing logic, it is not identical with it. As Piaget and Lévi-Strauss have argued, the art of kindling fire, inventing bows and arrows, smelting iron, producing pottery, or building houses all produce logic in its most elementary, preverbal form. As Ryle (1956) put it, informal logic precedes formal logic, just as “to know how” precedes “to know that.” In any case inventing and dealing with artifacts, including the invention of language and of symbols, lies at the bottom of the Intellect, as it lies at the bottom of logic.

**The Power of Logic**

The two quantifiers ∀ and ∃ thus corroborate on strictly logical grounds Leibniz’ thesis that all knowledge falls into two categories, viz. ordinary factual truth a posteriori and rational truth a priori, or as Kant put it in his *Critique of Pure Reason* (A 836), historical knowledge ex datis and rational knowledge ex principiis.

What Kant could not know in 1781 is Wittgenstein’s (1922) path-breaking discovery that knowledge ex principiis is founded in the sixteen truth-functions (*Tractatus Logico-Philosophicus* 5.101), each of which is defined by four truth-possibilities, viz. a&b (++), a&~b (+−), ~a&b (−+), and ~a&~b (−−), all four of which must be satisfied in order to vali-
date a given function. No less important, “The structures of propositions stand to one an-
other in internal relations” (Tractatus 5.2), i.e., every function is convertible into its con-
verse, joint denial, and dual, which define each other on strictly analytical grounds:

<table>
<thead>
<tr>
<th>Normal Form:</th>
<th>Converse:</th>
<th>Joint Denial:</th>
<th>Dual:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a v b.</td>
<td>~a</td>
<td>~b.</td>
<td>~(a ↓ b).</td>
</tr>
<tr>
<td>a ↓ b.</td>
<td>~a &amp; ~b.</td>
<td>~(a v b).</td>
<td>~(~a</td>
</tr>
<tr>
<td>a &amp; b.</td>
<td>~a ↓ ~b.</td>
<td>~(a</td>
<td>b).</td>
</tr>
<tr>
<td>a</td>
<td>b.</td>
<td>~a v ~b.</td>
<td>~(a &amp; b).</td>
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Like the quantifiers and modal logic, the four symmetric functions are convertible
into each other salva veritate by adding various negations (~). Each function is therefore
clearly recognizable only if it is not modified by negations. At the same time conversion
to conjunction (in bold letters) reveals the truth-values that determine each truth-function.
As illustrated by de Morgan’s law, (a v b) does not allow both a and b to be negative, i.e.,

(a v b) = ~(~a & ~b) = ~a | ~b = ~(a ↓ b).

Accordingly, ~a rules out ~b, and ~b rules out ~a, i.e., ~a posits b, and ~b posits a (modus
ponens). Conversely, using the Sheffer function (a | b), a rules out b, just as b rules out a
(modus tollens), i.e.,

(a | b) = ~(a & b) = ~a v ~b = ~(~a ↓ ~b).

Finally, the Nicod function (a ↓ b), i.e., “neither a nor b,” is self-explanatory:

(a ↓ b) = ~a & ~b = ~(a v b) = ~(~a | ~b).

By the same token conversion to the conjunctional form sorts out whether the con-
junction of a and b is either possible (P), necessary (N), or impossible (~P), which indi-
cate the truth-values that determine each function:

<table>
<thead>
<tr>
<th>a v b</th>
<th>P a&amp;b;</th>
<th>P a&amp;~b;</th>
<th>P ~a&amp;b;</th>
<th>~P ~a&amp;~b.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ↓ b</td>
<td>~P a&amp;b;</td>
<td>~P a&amp;~b;</td>
<td>~P ~a&amp;b;</td>
<td>~P ~a&amp;~b.</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
<td>~P a&amp;b;</td>
<td>P a&amp;~b;</td>
<td>P ~a&amp;b;</td>
</tr>
<tr>
<td>a &amp; b</td>
<td>N a&amp;b;</td>
<td>~P a&amp;b;</td>
<td>~P a&amp;~b;</td>
<td>~P ~a&amp;~b.</td>
</tr>
</tbody>
</table>

The four symmetrical functions thus form a tightly knit, well organized quaternary of
zero-form, converse, joint denial and dual, which, as Piaget has shown, is underpinned by
the most elementary forms of social action, moral judgment and rational thought.
Implication and Presupposition

What holds for the symmetrical functions also holds for the asymmetrical functions of implication and presupposition, which convert as easily into each other’s converse as they convert into the symmetrical functions, i.e.,

\[
\begin{align*}
& a \rightarrow b. = \sim a \leftarrow \sim b. = \sim(a \& \sim b) = a \mid \sim b = \sim a \lor b = \text{TFTT}, \\
& a \leftrightarrow b. = \sim a \rightarrow \sim b. = \sim(\sim a \& b) = \sim a \mid b = a \lor \sim b = \text{TTFT}.
\end{align*}
\]

Again the conjunctional form stands out as the most pregnant definition of each function: \(\sim(a \& \sim b)\) defines implication, just as \(\sim(\sim a \& b)\) defines presupposition.

By the same token their joint denial is easily found out: Non-entailment denotes randomness or accident (acc), and non-foundation denotes independence (ind):

\[
\begin{align*}
& a \text{ acc } b. = \sim (a \rightarrow b). = \sim (a \mid \sim b). = \sim(\sim a \lor b) = \text{FTFF}, \\
& a \text{ ind } b. = \sim (a \leftrightarrow b). = \sim (\sim a \mid b). = \sim(a \lor \sim b). = \text{FFTF}.
\end{align*}
\]

We have thus identified two functions which are absent in most logic books but which are fundamental for defining implication (entailment) and presupposition (foundation): Entailment reduces randomness, just as foundation reduces independence. It is by putting each calculus into the quaternary context that its function is made transparent. In terms of Luhmann (1995) entailment and foundation reduce complexity, whereas pure randomness would result in a chaos in which everything is in flux, just as pure independence would result in Parmenidian isolation and stagnation.

Like the functions of the symmetric quaternary, the functions of the asymmetric quaternary are based solely on internal coherence rather than correspondence with external facts. Accordingly, they are based on validation rather than verification -- an insight which is missing in Popper’s advocacy of falsifiability and piecemeal amelioration. To be sure Popper had a point in emphasizing that facts can falsify but not verify apodictic -- as he calls it, universal -- truth. What he failed to realize is that universal truth is amenable to validation by logic by putting it in the context of its quaternary, and even more poignantly, by transforming it into symmetric form, notably into disjunction and conjunction. Implication thus turns out to be equivalent to \((a \mid \sim b). = \sim(a \& \sim b)\)” just as presupposition is equivalent to “\((\sim a \mid b). = \sim(\sim a \& b)\)” both of which are prime examples of William James’s “radical empiricism.”
**Entailment versus Foundation**

Obviously entailment and foundation, causality and hierarchy are pivotal in constructing theory. At the same time the asymmetry that inheres in implication and presupposition invests human thought with a degree of flexibility that refutes the rigid determinism of the 17th and 18th century. While it is true that every cause implies an effect, it is no less true that every effect allows for a variety of causes, including unknown causes, as indicated by the formula

\[(a \rightarrow b) = \neg(a \& \neg b) = (a \mid \neg b) = P\, a,\, N\, b. = \diamond a; \Box b.\]

Likewise, while every superstructure presupposes a basis, it is no less true that a variety of superstructures can emerge from a given basis, including the inadequacy or lack of a superstructure, as indicated by the formula

\[(a \leftarrow b) = \neg(\neg a\&b) = (\neg a \mid b) = N\, a,\, P\, b. = \Box a, \diamond b.\]

The preceding considerations refute the prejudice that logic is either merely tautological and hence irrelevant, or that it is “imperialistic” and strangles freedom of thought. Quite to the contrary, it is the essence of asymmetric functions that for all their formal rigor they are open-ended. Thus determinate effects may involve a variety of causes, just as necessary conditions allow for a wide variety of emergent superstructures.

Thus history reads causality backwards, looking for diverse probable causes to explain given historical events such as the declaration of independence or the outbreak of the first World War. Looked at more closely, reality is inherently over-determined – a point made by Lenin and taken up by Althusser in refutation of monocausal determinism.\(^{12}\) As Hegel put it, causal linearity must yield to multiple circularity.

On the other hand, the foundational relationship is pivotal for sociological theory. Rather than getting lost in the endless struggle of all against all, Rousseau realized that social inequality originates with the successive differentiation and increasing complexity of the division of labor, private property and political power. The same paradigm informs Hegel’s *Philosophy of Law* and Auguste Comte’s *fascicule fondamentale* on *Politique positive* as well as Herbert Spencer’s trichotomy of sustaining, distributive and regulating functions. All of these path-breaking works relate the elements of their systems with
foundational relations which result in clear-cut hierarchies as reflected in Marx’s notion of basis and superstructure.

From a mere aggregate of elements which, like grains in a heap of sand, are freely interchangeable, there now emerge well-ordered hierarchies with an ascending order of complexity, novelty and specificity, as envisioned by Comte and corroborated by Parsons’ (1966) dual dichotomies of condition and control, energy input and information. What constitutes the strength of Marx’s and Engels’ “historical materialism” is therefore the legacy of Rousseau and St.-Simon, who rightly claimed Comte’s *fascicule fondamental* as his intellectual property.

In any case the criterion for strict sociological theory lies in the adequacy of its categories as well as in the foundational function that integrates them into a coherent system. The inductionist belief in the sequence from facts to empirical generalizations is thus reversed. Rather than on correspondence and the piecemeal accumulation of knowledge analytical theory is founded in coherence, i.e., on the construction of a conceptual framework that derives its validity from its internal consistency. As Kant (A 834) put it,

… that which we call science … is not formed in *technical* fashion, in view of the similarity of its manifold constituents, … but in *architectonic* fashion, in view of the affinity of its parts through which the whole is first made possible.

Yet while rational truth *transcends* factual truth, it does not dispense with it. Unlike facts, truth-functions are not given by mere observation nor do they inhere in reality. Rather they wait to be chosen at the individual thinker’s risk. Rational truth is therefore not immutable nor is it exempt of error, as assumed by Plato. Invariably it is the product of the individual Intellect, and as such subject to error and amendment.

**Validation versus Verification**

Obviously logic plays a much more fundamental role in the construction of theory than empiricism is ready to admit. In terms of Popper’s *Logic of Scientific Discovery* there is no way in which universal statements can be judged except by falsification. Unfortunately Popper (1962, 1979) had little to say about how strict science comes about except that it implies piecemeal *Conjecture and Refutation.*
Apparently unbeknownst to Popper the decisive step to show how existential statements ($\exists(x)$) turn into universal statements ($\forall(x)$) was made by Gottlob Frege’s (1893) distinction between unsaturated (algebraic) variables and saturated (substantive) arguments: To be true substantive arguments must satisfy the function that connects them, a requirement that does not hold for algebraic variables such as $a$ and $b$. For example, to validate the conjunction “Butterbread,” the substantive, “saturated” arguments “Butter” and “Bread” must satisfy the truth-values which define conjunction, i.e.,

$$a\&b. = N a\&b; \sim P a\&\sim b, \sim P \sim a\&b, \sim P \sim a\&\sim b. = TFFF.$$

Accordingly,

$$BU\&BR. = N BU\&BR; \sim P BU\&\sim BR, \sim P \sim BU\&BR, \sim P \sim BU\&\sim BR. = TFFF.$$

By contrast, let us imagine we have to choose between “either a cup of tea or a cup of coffee, but not both,” i.e.,

$$a \mid b = \sim P a\&b, P a\&\sim b, P \sim a\&b, P \sim a\&\sim b. = FTTT,$$

which is satisfied by

$$TEA \mid COF. = \sim P TEA\&COF, P TEA\&\sim COF, P \sim TEA\& COF; P \sim TEA\&\sim COF.. = FTTT.$$

An intricate mechanism thus rules analytical theory, which subjects it to its own type of test, viz. validation rather than verification. Whichever truth function we choose, in order to be valid the arguments must satisfy all four of its truth-values.

As Quine (1987: 215) commented on Tarski’s recursive method,

> What Tarski does define recursively is satisfaction of a sentence by an object or sequence of objects as values of its free variables.

In terms of Frege, formal tautologies are converted into substantive truths by substituting unsaturated (free) variables with saturated (bound) variables. In terms of Quine,

> Atomic sentences are neither true nor false; ‘Fxy’... is true of certain pairs of objects as values of the free variables ‘x’ and ‘y’, and false of others. It is only when all the variables in a sentence have been bound by quantifiers that we have a closed sentence, true or false (Quine 1987, p. 215).

While Quine was critical of the Kantian *a priori*, he accepted the dualism of coherence versus correspondence theory, seemingly unaware that the new terminology is but a modern substitute for Leibniz’ dichotomy of rational and factual truth. While we may agree with Quine’s critique of Kant’s interpretation of truth *a priori*, we still hold that his *syntheses a priori* are marked by rational truth, coherence, and apodicticity.
At the same time Quine’s reference to “atomic sentences” – which according to his own formulation are neither true nor false and hence do not constitute sentences – alerts us to another fundamental insight that goes back to Bergson, William James and Piaget (1970): Discursive logic is founded in a preverbal, “sensorimotor” stage. Just as language is not inborn, neither is logic. Invariably “knowing how” precedes “knowing that,” just as informal logic precedes formal logic and, as Tarde and Bourdieu have argued, imitation, respectively habitus precedes calculated rational action.

**Center and Periphery**

In the light of the preceding analysis it is obvious that far from being merely tautological logic plays an eminent cognitive role: What makes science possible is the validation of substantive content by the truth function that connects them. Interestingly, while Quine dismisses the notion of truth a priori, he (1982) distinguishes between center and periphery, viz. a hard core of persistent truths, which are surrounded by a soft periphery of contingent truths that wait to be incorporated into the core.

A fundamental difference thus exists between core and periphery: The center is apodictic and analytical; the periphery, conjectural and factual. Since apodicticity and contingency are irreducible to each other, science faces an ineluctable dilemma: either to adapt logic to experience, as do the inductivists, or to adapt experience to logic, as do the rationalists. On the one hand, for the empiricist facts are sacred and resist revision: For him the sun rotates around the earth, the whale is a fish, etc. On the other hand, once a coherent system of rational truths has been developed, change is possible only at the price of reconstruction from scratch or of fragmentation at the expense of coherence.

However, as Thomas Kuhn (1970) has argued against Popper, a disproved paradigm is not necessarily discarded. Rather, it may continue to be accepted so long as it serves practical purposes (e.g., the Ptolemaic system after Copernicus, or Newtonian physics after Einstein) and, most important, so long as it is not replaced by a more convincing system that may be a long time in the making even though the old system is discredited.

Moreover, as W.V. Quine (1982:2) has pointed out, “a thick cushion of indeterminacy” separates the center from the periphery and protects it from the fallibility of the senses while it protects the periphery from the “imperialism of reason.”
In sum, knowledge is dual. Like woof and warp it combines the continuous flux of contingent experience with discrete ideal constructs. Accordingly, knowledge develops on two different stages and in two different tempi. On the one hand—as it were, on the front stage—empirical knowledge cumulates like the sediments of a river, which depend on contingent external factors. On the other hand—as it were, on the backstage—analytical theory is based on ideal constructs, which gain momentum as they grow in coherence, simplicity, and completeness, none of which is based on observation. Obviously the two types of knowledge relate not only like core and periphery, but also like correspondence and coherence and like factual truth \textit{a posteriori} and rational truth \textit{a priori}.

**Scientific Revolutions and Normal Science**

At a first glance Thomas Kuhn’s (1970) dichotomy of revolutionary and normal science seems to fall in line with Quine’s dichotomy of center and periphery. However, this is not the case. Looked at more closely, normal science is as much a part of the core as is revolutionary science. It is therefore not identical with factual truth – a category seemingly inexistent for Kuhn, who compares revolutionary science with religious frenzy, and normal science, with mindless dogmatism. Our own interpretation would be that revolutionary science is dialectical, i.e., driven by the intermittent pressure of facts on logic and of logic on facts, whereas normal science is pledged to a fixed pattern. As Wolfgang Stegmüller (1976, 1986) has argued, no revolution is complete without subsequent consolidation. Accordingly, no scientific revolution is complete from the start. It is complete only after an extended period of maturation and hard work, which should not be downgraded as mere technicality.

Kuhn’s work on \textit{Scientific Revolutions} has shaken the widespread belief in the continuous accumulation of knowledge – an insight that goes back to Hegel’s notion of “dialectical leaps” and more recently, to Bachelard’s notion of \textit{ruptures}. What Kuhn has failed to note is the fundamental disparity of factual and logical truth, whose interaction accounts for the dynamics of science. The first is contingent, but cumulative; the second, exclusive and noncumulative. Science advances in leaps and bounds because it fluctuates between the piecemeal observation of facts and construction from scratch.
Underdetermination and Overdetermination

The dichotomy of truth *a priori* and *a posteriori* sheds light on another fundamental fact: Factual truth is indexical and atomic; by contrast, logical truth is imaginary and selective, i.e., it selects only facts that are values over its variables. As Quine (1953:13) put it,

A theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmation made in the theory is true.

Thus, theory does not merely transcend reality. It actively selects and rearranges our knowledge of it because rather than on sensual perception it is based on truth functions. By the same token, theory is predicated on intension, which defines sets, rather than on extension, which measures quantity. Again, Quine’s dichotomy of center and periphery is illuminating:

While a conflict with experience at the periphery occasions readjustments in the interior of the field, the latter is so underdetermined by ... experience that there is much latitude of choice as to what statements are to be reevaluated in the light of any single contrary experience... Total science, mathematical and natural and human, is similarly but more extremely underdetermined by experience. The edge of the system must be kept square with experience; the rest ... has as objective the simplicity of laws (Quine 1953:42).

Quine thus addresses another fundamental quality of analytics: Besides being apodictic and selective, it is underdetermined. While it deals with reality, it does not reflect it, but makes it transparent by reducing it to structure. By contrast, reality, and most prominently historical reality, is always overdetermined and by the same token opaque. For example, as Lenin argued against Plekhanov’s *nomothetic* positivism, the outbreak of World War I and the Russian revolution had multiple causes, none of which would by itself have been sufficient to trigger the war or the revolution. In other words, while none of the factors was 100% effective, when combined it took more than 100% to bring the war or the revolution about. Invariably reality constitutes a totality that is multifaceted and inexhaustible and therefore requires structural reduction to become transparent.

A fundamental disparity thus separates “real,” historical totalities from abstract, non-temporal systematicity: the first, overdetermined; the second, underdetermined; the first,
harboring historicism; the second, harboring structuralism as the two opposite modes of seeing the world.

**The Fallacies of Middle Range Theory**

Obviously analytical theory is fundamentally different from empirical description. As David Willer has argued, what distinguishes empiricism from theory is the lack of stringent models, no matter whether it deals with physics, society or ethics. “Theories of the middle range” are therefore no panacea to salvage sociological theory. To be sure, there is nothing wrong with theories of the middle range provided they go beyond empirical generalization and construct models that are founded in systematicity rather than empirical description. As David Willer (1987: 266) put it,

> In the world we can observe objects and events, but we cannot observe the relations among them. Any theoretical model solves that problem by building a world of related abstract objects and events, which can be shown to be similar to some objects and events in the world.

Willer (1987: XVII) goes even farther:

> If Einstein is right that theory is the free creation of the human intellect, then theory is not a simple reflection of the world... At issue is a faulty conception of theory now current in sociology, which has made it impossible to formally address important issues of social relationships and social structure.

In other words, social and historical narrative turns into strict theory only if actual, observable relations between individual actors are replaced by logical functions which convert contingent totality into analytic systematicity. Robert Merton (1968: 51) had therefore a point when he lashed out against “grand theory” as “the 20th-century equivalent of the large philosophical systems of the past with all their suggestiveness, their architectonic splendor, and their scientific sterility,” but he overstepped his competence when he denounced all attempts to construct strict theory as “a premature and apocalyptic belief.” Like Schmoller and the historicist school before him he confounds totality with systematicity when he suggests to “look toward progressively comprehensive sociological theory which gradually consolidates theories of the middle-range so that they become special cases of more general formulations.”
Merton (1968: 69) thus ends with a sour note for which he has himself to blame:
“We have many concepts, but fewer confirmed theories; many points of view, but few theorems; many ‘approaches,’ but few arrivals,” -- not because theories of the middle range are inherently unfit for theory, but because he fails to transcend piecemeal empirical generalization. As Carl Menger (1883) concluded in his trenchant critique of Schmoller which triggered the Historismusstreit,

Nothing will be achieved so long as “exact” and “historic-realistic” science are confused. Theory must transcend empiricism in order to be theory at all.

In the last analysis theory is a framework of ideal constructs which, like longitudes and latitudes, do not mirror reality but serve as instruments to come to grips with it.

May 18, 2003
Endnotes

1 Also note the work of Georges Gurvitch, who is not a Marxist, but is close to Marxian dialectics.

2 For a forceful argument for sociology as a strict science see Jonathan Turner (1993), Classical Sociological Theory. A Positivist Perspective, who sees in Herbert Spencer “the forgotten theoretical Giant” who set the parameters for a strict positivist social theory that allows to integrate the work of Durkheim, Marx, Max Weber and Mead into a consistent system.

3 Ironically the same holds for theology and for its secular substitute.

4 By the same token, the third International differs fundamentally from the second International, which was politically inept.

5 Also the prophet Amos (5,24) comes to our mind with his famous commandment: “But let justice run down as waters, and righteousness as a mighty stream.”

6 It thus elucidates why Marx had no taste for justice because it inherently obviates the dictatorship of the proletariat as of any other faction. Quite consistently, Lenin opted for the dismissal of the Constitutional Assembly and the brutal abuse of state power.

7 One need therefore not be surprised by the superiority of Anglo-Saxon democracy.

8 Far from being isolated abstractions meant to replace nomothetic theory, Weber’s famed ideal types thus turn out to capture the parameters of action and of choice.

9 As Peter Caws (1988) has put it, structuralism represents “the art of the intelligible.”

10 Popper thus lags far behind Peirce’s notion of ”abduction,” which is predicated on signs and calculi rather than on factual Conjectures and Refutations (Popper 1962).

11 Accordingly, Carnap (1947) distinguishes between logical truth (L-truth) and factual truth (F-truth).

12 A view of which the later Engels and his followers Kautsky and Plekhanov were not at all free.

13 By the same token, Max Weber (1970, p.24f.) distinguishes between traditional action” that is determined by habituation,” and zweckrational (purposive) action “that is determined by the actor’s own rationally pursued and calculated ends.”
The periphery thus stands for factual statements, or F-truths *a posteriori*, and the center, for apodictic statements, or L-truths *a priori*.

Fragmentation thus serves simplicity at the price of coherence and systematicity, helping us to know more and more about less and less.

Popper's (1972, ch.6) dichotomy of "the bucket and the searchlight" parallels the same dichotomy of factual and logical truth while he has little to say about the difference between truth *a priori* and *a posteriori*. 
THE LOGICAL FOUNDATIONS
OF ANALYTICAL SOCIOLOGY

The Dual Composition of Logic

The preceding chapter has shown the intricate link of analytical theory with Ontology. Once the latter is freed of its Platonic legacy it is rehabilitated as the precursor of modern structuralism. As the comparison of implication \((a \rightarrow b)\) and presupposition \((a \leftarrow b)\) has shown, the power of logic is all-pervasive. Theory has no longer to wait for mathematics to become a strict, “exact” science, and what recent discussion has touted as a “linguistic turn” turns out to be the turn to logic as it is bound up with language as its carrier.

At the same time the notion of “applied logic” raises doubts because it reflects a deeply ingrained Platonism in our thought. While there exists indeed such a thing as “pure” (formal) logic, the latter is a derivative of the former, as the pragmatist revolution of “radical empiricism” has made clear. In order to emerge formal logic presupposes ordinary language, i.e., Speech. Rather than founding language in logic, the reverse is the case, i.e., Speech represents the primal branch of semiology from which formal logic is distilled. In fact, every identification implies the denial of its negation according to the formula

\[(a = a) \equiv \neg(a = \neg a). \rightarrow \neg(a = b). = (a = b) = FFFF.\]

The same holds for the constitution of language in early childhood, a tack pursued by Piaget and Lévi-Strauss, who noted the contrast of opposites in constituting the child’s Speech: Just as he has to distinguish turning right from turning left, the child starts discerning reality by distinguishing warm from cold, big from small, push from shove, friend from foe etc., even before acquiring language. In fact humans with the higher animals share all these elementary distinctions.
In every case human speakers and animals are equally unaware of the four truth-values that define every truth-function, and yet they think accurately. In the upshot Erkenntnistheorie reduces to what Kant might have called “transcendental Logic” – the term actually used by Husserl (1929) in contrast to formal Logic. The latter produces pure Logic, but blocks cognition because it works with anonymous variables. It is by using “saturated,” substantive variables that “transcendental Logic” produces truth.

**Truth-Conditions and Truth-Functions**

A clear distinction must therefore be made between arguments and functions. The first are substantive and refer to reality; the second are syntactic and are defined by their truth-values. Arguments and functions may therefore alter independently, as the various combinations of bread and butter, tea and coffee etc. easily show. What is less realized is that there exists an important difference between truth-functions, on the one hand, which connect substantive, “saturated” variables, and purely syntactical truth-conditions, on the other hand, which work with “unsaturated,” algebraic variables and block cognition – the dual reason why positivist logistics and Kantian Erkenntnistheorie are equally blind to the link that underlies all logic as well as all cognition.

It is therefore all the more remarkable that Wittgenstein (TLP: 4.01) chimed in with Kantianism. As he put it, "The proposition is a model of reality such as we conceive it." Whatever the relations between "things in themselves" and the abstruse juggling with *noumena*, it is the subsumption of substantive arguments under a truth function which constitutes logical, or rational truth *a priori*. As Wittgenstein (TLP 4.463) put it, "The truth-conditions of a proposition determine the range that it leaves open to the facts." It is the subsumption under truth-conditions which “makes strict science possible” while avoiding Kant’s convoluted notion of the *a priori*.

**Logical Quadrants**

The preceding analyses give a new answer to the Kantian question, How is science possible? Science transcends empirical truth if its propositions satisfy the *Wahrheitsbedingungen* of a truth function, i.e., as Wittgenstein put it (TLP 4.463), "*Die Wahr-"
heitsbedingungen bestimmen den Spielraum, der den Tatsachen durch den Satz gelassen wird." (The truth-conditions of a proposition determine the range that it leaves open to the facts.)

The question then arises how pure, seemingly mathematical logic is constituted. The answer is that there are exactly sixteen truth functions, which are ruled by a strict regime of consistency, simplicity (parsimony) and completeness which makes all truth functions convertible to one another so long as their truth-values are maintained.
THE FIRST QUADRANT: SYMMETRY AND COMMUTATIVITY

Zero-Form, Joint Denial, Converse, and Dual

Let us turn to the first of the four quadrants, which is the one that is most broadly known and easiest to interpret. It is characterized by two distinctive qualities: symmetry and commutativity. Most important, a strict order rules among the four functions, each of which relates to the others either as denial, converse, or dual.

(1) For obvious reasons, global, joint denial is the most elementary function of logic that divides truth functions into contradictory pairs which mutually define each other:

$$A \lor B. = \sim(A \downarrow B),$$
$$A \downarrow B. = \sim(A \lor B),$$
$$A \& B. = \sim(A \mid B),$$
$$A \mid B. = \sim(A \& B).$$

(2) Along with joint denial, there exists a converse to every truth function, which is marked by the negation of its arguments, i.e., by dual partial denial:

$$A \lor B. = \sim A \mid \sim B, \text{ known as modus ponens.}$$
$$A \mid B. = \sim A \lor \sim B, \text{ known as modus tollens.}$$
$$A \& B. = \sim A \downarrow \sim B, \text{ known as modus ponens.}$$
$$A \downarrow B. = \sim A \& \sim B, \text{ known as modus tollens.}$$

(3) Thirdly, we may combine global, joint denial and dual partial denial, which results in the dual, as known as De Morgan’s law, viz.

$$A \lor B. = \sim(\sim A \& \sim B).$$
$$A \& B. = \sim(\sim A \lor \sim B).$$

The dual is not limited to alternation and conjunction. It also holds for joint denial (\downarrow) and incompatibility (\mid):

$$A \downarrow B. = \sim(\sim A \mid \sim B),$$
$$A \mid B. = \sim(\sim A \downarrow \sim B).$$

We thus arrive at a stringent schema of mutually convertible truth functions:
Table 1
Zero-Form, Joint Denial, Converse, and Dual

<table>
<thead>
<tr>
<th>Zero Form</th>
<th>Joint Denial</th>
<th>Converse</th>
<th>Dual</th>
</tr>
</thead>
</table>

The consistency, perspicuity and stringency of the regime which rules the four functions is self-explanatory. Whatever the complexity of its form, each function is reducible to its normal form which presents the meaning of the function in the most straightforward and simplest form.

The first column shows each truth function in its pure, or zero-form, i.e., unencumbered by global or partial denial. For its purity and transparency we may also call it the normal form. It is also the form that is closest to everyday language.

The second column connotes the global, or joint denial of the normal form. It tells us what the function rules out. As it were, it sorts out the bad apples without saying much about the good apples.

The third column represents the converse of the zero-form. Unlike joint denial, it is specific about the arguments which it negates and thus spells out the logical implications of the normal form. For example, while the global denial

\[ A v B. = \sim (A \downarrow B), \]

looks somewhat contrived, the converse is forthright and explicit; it easily carries over to the dual, e.g.,

\[ A v B. = \sim A \mid \sim B. = \sim (\sim A \& \sim B). \]

Finally, the fourth column conveys the dual to the zero-form. It is known as De Morgan's law and is broadly used for the mutual conversion of the conjunctive and the disjunctive function. Otherwise, it adds little to what is already spelled out by the converse, e.g.,
A & B. = ~A ↓ ~B,
which reads “neither non-A nor non-B.” In every case, zero-form, denial, converse and dual define each other and constitute a self-explanatory, closed logical quadrant.

The preceding analyses have important implications:
(a) Whatever its complexity, any logical matrix is reducible to zero-form, which reveals its syntactical meaning.
(b) Any change beyond the conversion rules changes the meaning of the truth function and thus implies a switch to a different truth function with a different syntax.
(c) In spite of its rigor, logic invites *Ideenvariation* (Husserl 1913), inventiveness and creativeness. Like the arts, it is intellectually liberating and sensitive to elegance.
(d) Each function exists in its own right. While functions are convertible, they are not reducible to each other.

**The Pitfalls of Formal Logic**
The previous analysis of pure functions regardless of substantive content reveals the power and flexibility that inheres in the functions as a necessary, but not sufficient condition of human thought (Frege 1921): Pure logic produces schemata, but not truth. To produce the latter, and that is, to convert schemata into propositions, substantive arguments must complement the functions.

The cognitive role of logic is obscured and the syntactical meaning of the functions is outright lost if logic is reduced to a mere technique of shuffling truth-values over a universe which is reduced to "p" and "q." Whether a truth function is applicable only a glance at the intended meaning and the arguments can tell. To be fully understandable, syntactic meaning must combine with substantive meaning.¹

The emphasis on substantive truth implies the switch from formal, algebraic logic to substantive, applied logic in tandem with the switch from propositions to terms. As the discussion of sodium and chlorine has shown, terms rather than propositions are the hub of logic. In fact switching from propositional logic to terminal logic will spare as statements such as, e.g., "If Paris is the capital of China, squares are round" (Quine 1982, p.24).²
Far from being exemplary, statements of this sort represent a misuse of logic. They easily collapse if the propositions are substituted with terms: Even if it were true, e.g., that "Paris is the capital of France," and that "Squares are rectangular," it would still not be true -- and that is what the schema "If p, q" states -- that Paris implies squares. Nor does the argument fare any better if the underlying matrix is reinterpreted as "\(~p \rightarrow \sim q\)\), whose zero-form, "p \leftarrow q\), is equally inapplicable. Squares do not presuppose Paris as a necessary condition any more than they are a sufficient reason for it. Obviously, the two have nothing in common. It thus stands to reason that neither implication nor presupposition apply.

The question still remains how a true statement about Paris and about squares should look like. The answer is: "It is not true that Paris is the capital of China or that squares are round." We have no difficulty to identify the zero-form which underlies this statement:

\[ \sim (p \lor q) = \sim (-p \mid \sim q) = \sim p \land \sim q = p \downarrow q, \]

which reads: "Neither is Paris the capital of China, nor are squares round." The point is that the use of "if" in the chosen example results in a nonsensical statement because it runs counter to the implied meaning of the statement, which is, "if someone is ignorant enough to believe that Paris is the capital of China, he might as well believe that squares are round."

**Formal and Substantive Logic**

Our criticism of misapplied logic does not stop here. Our case can be argued on strictly formal grounds as well. Granting that arguments p and q are both false, nothing warrants strict implication (\(\rightarrow\)) as the pertinent truth function. Of the sixteen truth functions no less than eight allow for the combination of \(\sim p\) and \(\sim q\), viz.

\[
\begin{align*}
p \downarrow q & \quad p \mid q. \\
p \rightarrow q & \quad p \leftarrow q. \\
p \neq q & \quad p \leftrightarrow q. \\
N \sim p, P q v \sim q & \quad P p v \sim p, N \sim q.
\end{align*}
\]

The point is that nothing warrants the use of isolated truth-values. Without exception it takes all four truth condition, i.e., ++, +-, -+, --, to define any of the sixteen truth-functions, which in the present case is indubitably FFFT.
Everyday Speech and Substantive Logic

The switch from formal logic to substantive logic and to the inclusion of all sixteen truth-functions enhances the scope and power of logic and reconciles it with ordinary, everyday Speech.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Formal and Substantive Logic: the First Quadrant</th>
</tr>
</thead>
<tbody>
<tr>
<td>A v B</td>
<td>A ⊃ B</td>
</tr>
<tr>
<td><strong>Inventory</strong></td>
<td><strong>Incompatibility</strong></td>
</tr>
<tr>
<td>Enumeration</td>
<td>Exclusion</td>
</tr>
<tr>
<td>A ↓ B</td>
<td>A &amp; B</td>
</tr>
<tr>
<td>&quot;Neither A nor B&quot;</td>
<td>&quot;Both A and B&quot;</td>
</tr>
<tr>
<td><strong>Summary Denial</strong></td>
<td><strong>Complementarity</strong></td>
</tr>
<tr>
<td>Rejection</td>
<td>Inseparability</td>
</tr>
</tbody>
</table>

In every case ordinary Language, i.e. Speech, combines Logic with substantive content:

- adjunction maps inventory, enumeration, contiguity, alternation, free choice etc.;
- joint denial maps summary denial, rejection, rebuttal, dismissal etc.;
- disjunction maps incompatibility, exclusion, decision, strict choice etc.;
- conjunction maps complementarity, inseparability etc..

In other words, while formal logic deals with algebraic arguments such as A, B, etc., each function is marked by distinctive syntactic meaning regardless of the semantic meaning of the arguments which it connects. Its formal character notwithstanding Logic is therefore distinctly different from mathematics. It is by no means a subset of mathematics, let alone of physics, as logistics and the Viennese Circle would contend.

On the other hand, while the four functions are convertible to each other, any conversion is legitimate only if the truth-values which define each function remain unaltered. To reduce logic to a few functions such as adjunction (v), conjunction (&), and implication (→) is therefore to curtail Logic to the point of impotence.
THE SECOND QUADRANT: ASYMMETRY AND IRREVERSIBILITY

The first quadrant of truth functions is followed by a second quadrant which is no less impressive for its consistency and self-sufficiency. Rather than on symmetry and commutativity it is predicated on asymmetry and irreversibility. By the same token its functions combine necessity of one of its arguments with open-endedness of the other argument.

Implication and Replication

As a standard procedure in propositional logic, implication and its converse, replication (i.e., presupposition) are subsumed under the common denominator of "conditionals," thus blurring the fact that the two functions are quite different. To be sure, nobody disputes that

\[ A \rightarrow B. = A \neg B. = \neg A \lor B. = \neg (A \land \neg B) \]

is not identical with

\[ A \leftarrow B. = \neg A B. = A \lor \neg B. = \neg (\neg A \land B). \]

Confusion sets in if the sequence of the arguments is reversed on grounds of proof theory. Arguing that

\[ A \rightarrow B. = \neg A B. = A \neg B \]

is equivalent to

\[ B \leftarrow A. = B \lor \neg A. = \neg B A. \]

implies a gross fallacy, which becomes apparent when we translate the two schemata into ordinary language. "A→B" connotes "A implies B", or more forcefully, "B is a necessary consequence of A". By contrast, its inversion, "B←A" reads "A presupposes B," or "B is a necessary condition for A," which leaves us with a perplexing contradiction: How can the necessary consequence of A be at once its necessary condition, and conversely, how can the antecedent of a necessary consequence be possible only if the consequent is already presupposed? The point is that in contrast to formal, allegedly “mathematical” Logic substantive, material Logic does not allow for inversion, i.e.,

\[ \neg A B. \neq B \lor \neg A. \]
**Intensio Directa and Intensio Indirecta**

The problem does not lie with the imprecision of ordinary language, which remains remarkably truthful to reality. Nor is the problem to be attributed to logical indeterminacy. Rather, it lies with the improper use of truth-values. To illustrate our point, let us substitute "cause" (CAU) for A, and "effect" (EFF) for B. Then it is obvious that

$$\text{CAU } \Rightarrow \text{EFF. } = \sim (\text{CAU } \& \sim \text{EFF). } = \text{ CAU } \mid \sim \text{EFF},$$

i.e., "Cause entails effect as a necessary consequence" is equivalent to "It is not true that there is a cause and no effect," and to "The assumption of a cause excludes the denial of an effect." Wherever there is a cause, there must be an effect.

At the same time, the inversion "EFF $\leftarrow$ CAU" is not bare of sense: Awareness of a cause does indeed presuppose awareness of an effect. While both statements deal with the same *Sachverhalt*, CAU $\Rightarrow$ EFF addresses reality *intensio directa* whereas "EFF $\leftarrow$ CAU" addresses it *intensio indirecta*. The two matrices are therefore quite different in character: The first is a statement, the second, an inference which presupposes some material statement. Obviously inversion of the arguments is not permissible for the second quadrant because it confuses statements *intensio directa* with inferences *intensio indirecta*.

In order to avoid confusion, let us mark off the direct mode with a double-shafted arrow ($\Rightarrow$), and the indirect mode, with a single-lined arrow ($\rightarrow$) and quotation marks to denote *intensio indirecta*. In this manner A $\Rightarrow$ B denotes necessary consequence, while "B $\leftarrow$ A" denotes presupposition inferred from it, i.e.,

$$A \Rightarrow B. \rightarrow \ "B \leftarrow A".$$

Likewise, A $\leftarrow$ B denotes foundation, or necessary condition, while "B $\rightarrow$ A" denotes strict implication inferred ($\rightarrow$) from the foundational formula, i.e.,

$$A \leftarrow B. \rightarrow \ "B \rightarrow A".$$

In sum, double-shafted arrows denote objective statements. By contrast, single-shafted arrows denote inference derived from the former. A sharp distinction must therefore be made between the direct and the indirect mode, and by the same token, between "substantive" logic, which works with saturated variables and produces truth, and formal, algebraic logic, which works with unsaturated variables and does not produce truth.$^3$
Entailment and Necessary Consequence

To illustrate our case, let us substitute causality for strict implication. Then it is evident that once there is a cause (CAU), there must be an effect (EFF), i.e.,

\[ \text{CAU} \Rightarrow \text{EFF} = \neg(\text{CAU} \& \neg \text{EFF}) = \text{CAU} \mid \neg \text{EFF}. \]

At the same time, while inversion of the arguments is not permissible, conversion without changing the sequence of the arguments is legitimate: The equation

\[ \text{CAU} \Rightarrow \text{EFF} = \neg \text{CAU} \iff \neg \text{EFF} \]

stays in the direct mode (with the sequence of terms unchanged) and makes good sense: If a determinate cause entails a determinate effect, eliminating the cause is a necessary condition for avoiding the effect. (Note that the absence of a cause is not a fact, but an a conjecture):

\[ \neg \text{CAU} \iff \neg \text{EFF} \neq \neg \text{EFF} \Rightarrow \neg \text{CAU}. \]

Obviously it would be odd to assume that an inexistent effect could entail an equally inexistent cause. In sum, truth-functional appearance to the contrary, the equation

\[ \text{CAU} \Rightarrow \text{EFF} = \text{EFF} \iff \text{CAU}. = \neg \text{EFF} \Rightarrow \neg \text{CAU} \]

is false both on material and formal grounds: Materially, it is nonsensical; formally, it violates the rule that forbids inversion of the functions of the second quadrant.

However, the inversion becomes meaningful if we switch from the direct to the indirect mode, i.e.,

\[ \text{CAU} \Rightarrow \text{EFF} \rightarrow "\neg \text{EFF} \Rightarrow \neg \text{CAU}" = "\text{EFF} \iff \text{CAU}". \]

It certainly makes sense to infer that awareness of an effect is the necessary condition for searching for a cause. Indeed, looking for a cause presupposes not so much an effect than the awareness of an effect.

The distinction between direct and indirect mode has far-reaching consequences for the perspicuous understanding of Logic, for direct, “material” implication is not identical with “strict” implication. The latter is strict, not on ontological grounds (viz. empirical evidence), but on grounds of being a logical derivate of other functions which are founded in direct evidence such as necessary consequence, necessary condition and, as we shall see shortly, randomness and independence, as well as conjunction, disjunction, etc.
**Foundation and Necessary Condition**

As already noted, the conditional truth function follows the same principles as entailment, its converse. To illustrate our case, let us assume that the law forbids people under 18 years of age to drink beer in public places. Then it is obvious that being under the age of 18 rules out drinking beer and that being 18 is a necessary condition for getting beer, i.e.,

\[ \sim \text{Age18} \Rightarrow \sim \text{BEER.} = \sim \text{Age18} \mid \text{BEER.} = \text{Age18} \Leftarrow \text{BEER.} \]

It is obvious from this equation that the counterfactual is the most effective way to test whether a conditional is true. By the same token, the distinction between the direct and indirect mode is as crucial for conditionals as it is for entailment. Inverting the sequence of the arguments would be tantamount to saying that drinking beer entails becoming 18 years old, or that abstaining from beer is a necessary condition for being under 18, i.e.,

\[ \text{Age18} \Leftarrow \text{BEER.} \neq \text{BEER} \Rightarrow \text{Age18.} = \sim \text{BEER} \Leftarrow \sim \sim \text{Age18}, \]

all of which border on the nonsensical. However, seeing people drink beer in public we may infer that they are 18 years of age, i.e., getting beer strictly implies, but does not materially imply, i.e., entail, being 18, i.e.,

\[ \text{Age18} \Leftarrow \text{BEER.} \Rightarrow \text{“BEER} \rightarrow \text{Age18.”} \neq \text{BEER} \Rightarrow \text{Age18.} \]

In every case, the distinction between the direct and indirect mode, i.e., between *intentio directa* and *intentio indirecta* accounts for the difference between material foundation (\( \Leftarrow \)) and strict implication (\( \rightarrow \)). As in the case of causality the distinction between the direct and the indirect mode is fundamental for understanding Logic – a requirement which most logic books do not meet, which reduce the difference between “bound” and “unbound” variables to a mere technicality.

At the same time the role of strict implication is easily demonstrated by the elementary fact that each of the sixteen truth-functions is convertible not only to its joint denial, converse and dual, but as Nicod and Sheffer have shown, to any other of the sixteen truth-functions *salva veritate*, however complex the formula. For example,

\[
\begin{align*}
A \lor B. &= (\sim A \rightarrow B. \& \sim B \rightarrow A. ) \& (\sim(\sim A \rightarrow B. ) \& \sim(\sim B \rightarrow A. ) ) . \\
A \& B. &= (A \rightarrow B. \& B \rightarrow A. ) \& (\sim(\sim A \rightarrow B. ) \& \sim(\sim B \rightarrow A. ) ) . \\
A \leftrightarrow B. &= (A \rightarrow B. \& B \rightarrow A. ) \& (\sim A \rightarrow \sim B. ) \& (\sim B \rightarrow \sim A. ) . \\
A \text{ opp } B. &= (A \rightarrow \sim B. \& \sim B \rightarrow A. ) \& (\sim A \rightarrow B. ) \& (B \rightarrow \sim A. ) .
\end{align*}
\]
Randomness and Independence

For good reasons implication stands at the center of propositional logic: It seems to be the mechanism which imparts necessity to our thought, whereas the denial of necessity seems foreign to strict thought and has met with scant attention. All that seems to be known about it in the work of Frege (1922) or Wittgenstein (1921) are the truth-values FTFF and FFTF and their identification as "A&~B" and "~A&B," which say nothing about their zero-form, i.e.,

\[ A \& \neg B. = \neg (\neg A \lor B). = \neg (A \Rightarrow B), \]

respectively,

\[ \neg A \& B. = \neg (A \lor \neg B). = \neg (A \Leftarrow B). \]

What the conjunctural and the disjunctural formulas do not bring out, the joint denial of necessary condition (foundation) and of necessary consequence (entailment) bring to the fore: To say that A does not entail B is to say that A has no impact on B and that B is accidental to A (A acc B). In sum, the denial of entailment implies randomness, coincidence, or chance, which concurs with Frege's and Wittgenstein's definition:

\[ A \text{ acc B.} = \neg (A \Rightarrow B). = \neg (A \upharpoonright \neg B). = A \& \neg B. = \text{FTFF} \]

In a similar fashion the denial of necessary condition defines independence: To deny that A is a necessary condition for B is to say that B is independent of A.

By the same token independence is the converse of randomness:

\[ A \text{ ind B.} = \neg (A \Leftarrow B). = \neg (\neg A \upharpoonright B). = \neg A \& B. = \text{FTTF} \]

While the two schemata, "A&~B" and "~A&B" are silent about reality, randomness and independence represent substantive, ontological categories. Together with entailment and foundation they constitute the second quadrant as a self-explanatory domain of logic.

Most important, Logic is no longer hooked up with determinism. The second quadrant reveals the truth-functional foundations of necessity. Longstanding beliefs to the contrary, necessity is not a metaphysical category, but a logical category, whose denial explains randomness and independence. As modern physics has fully brought out, indeterminacy and rupture, spontaneity and discontinuity rule the physical cosmos as well as human thought. In every case the functions of logic underlie our ontological categories as the primal instruments of cognition.
The Second Quadrant as a Whole

Except for the basic difference between symmetry and asymmetry, the second quadrant follows the same principles of mutual convertibility as the first quadrant.

Table 3
Asymmetrical Functions and their Equivalents

<table>
<thead>
<tr>
<th>Zero-Form</th>
<th>Denial:</th>
<th>Converse:</th>
<th>Dual:</th>
</tr>
</thead>
</table>

At the same time the asymmetric functions are convertible into symmetric functions, which are much easier to handle. Thus entailment and causality easily translate into, and are defined by ~A v B = A | ~B. = ~ (A & ~B), just as hierarchy and foundation translate into, and are defined by A v ~B. = ~A | B. = ~ (B & ~A):

Table 4
Conversion of Asymmetric Functions into Symmetric Form

<table>
<thead>
<tr>
<th>A ⇒ B</th>
<th>~ (A &amp; ~B)</th>
<th>~A v B</th>
<th>A</th>
<th>~B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ⇐ B</td>
<td>(B &amp; ~A)</td>
<td>A v ~B</td>
<td>~A</td>
<td>B</td>
</tr>
<tr>
<td>A accB</td>
<td>A &amp; ~B</td>
<td>~ (~A v B) = ~ (A</td>
<td>~B)</td>
<td></td>
</tr>
<tr>
<td>A ind B</td>
<td>B &amp; ~A</td>
<td>~ (A v ~B) = ~ (~A</td>
<td>B)</td>
<td></td>
</tr>
</tbody>
</table>

In this way we arrive at a neat picture of the functions of the second quadrant, which reveals its inner rationale while it is self-explanatory:
Table 5  
The Functions and Categories of the Second Quadrant

<table>
<thead>
<tr>
<th>A ⇒ B</th>
<th>A ⇔ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>= ~A v B = A</td>
<td>= A v ~B = ~A</td>
</tr>
<tr>
<td>TFTT</td>
<td>TFTT</td>
</tr>
</tbody>
</table>

**necessary Consequence**
- Entailment
- Causality

**necessary Condition**
- Foundation
- Hierarchy

<table>
<thead>
<tr>
<th>A acc B</th>
<th>A ind B</th>
</tr>
</thead>
<tbody>
<tr>
<td>= ~ (A⇒B) = A&amp;~B</td>
<td>= ~ (A⇔B) = ~A&amp;B.</td>
</tr>
<tr>
<td>FTFF</td>
<td>FFTF</td>
</tr>
</tbody>
</table>

**Randomness**
- Contingency
- Indeterminacy

**Independence**
- Fragmentation
- Leap, Rupture
THE THIRD QUADRANT: IDENTITY AND EQUIVALENCE

It is clear from the preceding discussion of symmetric and asymmetric truth functions that only the context of the quadrant provides the full picture of its functions. Let us now turn to the third quadrant, whose truth functions are quite different and have rarely if ever been treated as a self-explanatory unit. As we shall see, identity and equivalence are converse functions which along with their negations constitute a unit which is no less intricate than the first and second quadrants.

On the whole the third quadrant still waits to be recognized as a self-explanatory unit. On the one hand, equivalence (↔) and its denial, “contravalence” (>¬<), or polar opposites (opp), are treated on a par with the truth-functions of the second quadrant and are variously interpreted as "biconditional," "bisubjunction" (Lorenzen 1965: 33), or "mutual implication" (Quine 1982: 25), respectively as "exclusive disjunction" (Łukasiewicz, Bochenski 1964, Quine 1982: 18), or "bisubtraction" (Lorenzen 1965: 30).

By contrast, tautology (TTTT) and contradiction (FFFF) are rarely recognized as truth functions. For the most part, they are treated as synonymous with affirmation and negation (Wittgenstein TPL 5.101; Łukasiewicz; Bochenski 1962) and are subsumed under predicative logic. It is thus not surprising that the internal unity of the quadrant has gone widely unnoticed.

Nobody disputes that identity plays a key role in logic. Identity and non-identity, or difference, are the wellsprings of meaning and of thought in general. In addition, equivalence and polar opposites are the basis of mathematics and measurement. Nobody seems to have considered them components of a common quadrant.

Equivalence and Polar Opposites

At a first glance the third quadrant seems to replicate the symmetry of the first quadrant. However, a closer look on equivalence and opposition shows that the functions of the third quadrant are not merely symmetric, but reflexive and duplicative. Thus "A↔B" does not simply read, "A is equivalent to B, and B is equivalent to A", as this is the case with the functions of the First quadrant. What mutual implication, the so-called biconditional, really denotes is congruity not only of the two arguments (which explains their
reflexivity) but also of their negative complements, i.e., to say “A is congruent with B” implies “~A is congruent with ~B”:

\[ A \leftrightarrow B = (A \rightarrow B. \ & \ B \rightarrow A) \ & \ (~A \rightarrow ~B. \ & \ ~B \rightarrow ~A) \]
\[ = (A \leftrightarrow B) \ & \ (~A \leftrightarrow ~B) \]
\[ = (A&B) \leftrightarrow (~A&~B). \]

The same principles apply to polar opposites. Not only does A imply non-B, and non-B, A to ensure congruence, but non-A must also imply B, as B must imply non-A:

\[ A \text{ opp } B. = (A \rightarrow ~B. \ & \ ~B \rightarrow A) \ & \ (~A \rightarrow B. \ & \ B \rightarrow ~A) \]
\[ = (A \leftrightarrow ~B) \ & \ (B \leftrightarrow ~A) \]
\[ = (A&B) \leftrightarrow (B&~A). \]

**Tautology and Contradiction**

At the same time, an intricate calculus rules tautology and contradiction which veils their correspondence with equivalence and polar opposites. The puzzle starts when tautology is misconstrued as A=B, and accordingly, contradiction is misconstrued as A ≠ B. Instead, it is the statement that "A is not identical with B”, i.e., "A ≠ B," which is true no matter whether A or B are true or false. It represents the truth function with four positive truth-values, viz. TTTT.

In sum, non-identity is not contradiction. If A and B are different, they are non-identical, but not necessarily contradictory. Rather, to state that A and B are different is to state that they are each identical only with themselves and that identity with anything else is always false. Appearance to the contrary, the schema "A ≠ B" denotes identity and difference, i.e.,

\[ A \neq B. = (A \neq ~A) \ & \ (B \neq ~B). = (A \neq ~A) \ & \ (B \neq ~B) \]
\[ = A=A, ~A=\sim A, B=B, \sim B=\sim B, \]
which is always true, TTTT, whatever the values of A or B.

By contrast, contradiction produces four negative truth-values, FFFF. The meaning of contradiction is that it denies that A is not identical with non-A:

\[ A = B. = \sim (A \neq B). \ & \ \sim (A \neq ~A). \ & \ \sim (B \neq ~B) \]
\[ = (A = \sim A) \ & \ (B = \sim B). \]

Obviously, tautology and contradiction follow the same rationale as equivalence and polarity: congruence implies reflexivity and duplication. By the same token each function
of the quadrant is convertible to conjunctional and disjunctional form, as the following
table brings out:

<table>
<thead>
<tr>
<th>A ≠ B</th>
<th>A ↔ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTTT</td>
<td>TFFT</td>
</tr>
<tr>
<td>A=A, ~A=~A, B=B, ~B=~B.</td>
<td>(A↔B) &amp; (~A ↔ ~B)</td>
</tr>
<tr>
<td>A ⏐~A &amp; B ⏐ ~B</td>
<td>A ⏐~B &amp; B ⏐~A</td>
</tr>
</tbody>
</table>

Identity & Difference

Identity & Difference

<table>
<thead>
<tr>
<th>A = B</th>
<th>A opp B</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFFF</td>
<td>FTTF</td>
</tr>
<tr>
<td>(A = ~A) &amp; (B = ~B)</td>
<td>(A ↔ ~B) &amp; (B ↔ ~A)</td>
</tr>
<tr>
<td>~(A ⏐~A) &amp; ~(B ⏐~B)</td>
<td>A ⏐ B &amp; ~A ⏐~B</td>
</tr>
</tbody>
</table>

Contradiction

Contradiction

The Function of Identity and Difference: Set Theory

Looked at more closely, identification is not only the most basic of all truth functions,
because it is the mechanism that generates meaning; it is also the most radical of all func-
tions: Identity absorbs everything that is identical, just as non-identity repulses everything
that is not identical\textsuperscript{10}.

Moreover, finding out what is different and what is identical in two objects starts the
dual process of generalization (by looking for the qualities which are common) and of
differentiation (by specifying the qualities which are not common)\textsuperscript{11}. It is in this way that
sets and subsets are formed and that set theory originates.

It thus becomes clear that far from being limited to sterile tautologies and contradic-
tions, identification and differentiation is the hub of all thought, and that thought harbors
all the potential for innovation and creativity. It is thus not true, as Kant has argued, that
the predicate is contained in the subject. Invariably the common genus adds information
which reaches beyond its species. Invariably it involves judgment, or as Hegel put it: \textit{die
Anstrengung des Begriffs}. 

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A derivative, abstract “grammar” thus emerges from within ordinary language, which constitutes “pure” logic in the sense that it eliminates all substantive, “satiated” variables by replacing them with “unsaturated,” algebraic variables (Frege). The original life-world in which the individual acts, feels and thinks is thus supplemented by abstractions and generalizations which replace "knowledge by acquaintance" with "knowledge by description" (Russell 1914). Concepts replace percepts, as analytics replaces indexicality.\(^\text{12}\)

Identification and differentiation thus lie at the root of set theory, as the latter lies at the root of analytical theory. While it is true that sets and classes are extensional, there are no sets without descriptions which define them. Invariably extension presupposes intension, i.e.:

\[
\begin{align*}
\text{INT} & \iff \text{EXT}. \\
\neg\text{INT} & \iff \neg\text{EXT}. \\
\text{EXT} & \rightarrow \text{INT}.
\end{align*}
\]

**The Function of Equivalence and Opposition: Mathematics**

Much has been made of the fact that set theory founds mathematics, including the opposite attempts to reduce mathematics to logic (Frege 1893; Whitehead and Russell 1910) or to reduce logic to mathematics (Boole 1854; E. Schröder 1878). The truth is that mathematics presupposes, but does not reduce to, logic. As Boole (1854) realized, the laws of algebra are not reducible to logic, but he stopped short of acknowledging that mathematics presupposes logic, and in particular the functions of equivalence and opposition (contravalence):

\[
\begin{align*}
\text{LOG} & \iff \text{MATH}. \\
\neg\text{LOG} & \iff \neg\text{MATH}. \\
\text{MATH} & \rightarrow \text{LOG}.
\end{align*}
\]

Invariably, mathematics presupposes logic, and in particular equivalence and contravalence, which make mathematics possible. In a similar fashion polar opposites mark extremes such as hot and cold, young and old, and in particular, 0 and 1, which provide the parameters for scaling and measurement. Time and age, size and distance, weight and price, temperature and air pressure, are amenable to measurement by establishing scales. No matter how different the scales, as the differences between miles and kilometers, ounces and grams, etc. illustrate, they are all predicated on polar opposites.
In spite of their indispensability polar opposites, like equivalences, are transfinite. Just as the combination of long vs. short, high vs. low, and narrow vs. broad indicate the dimensions whose Cartesian product founds analytical geometry, i.e.,

\[ \text{Extension} = \text{Length} \times \text{Height} \times \text{Breadth} \]

the identification of similar parameters founds analytical social theory. To give the reader an idea of the logical roots of analytical sociology, it is easy to understand that the following parameters “determine the range that is left open to the facts:”

- **skills** determine the range of the *occupational structure*;
- **property** determines the range of the *property or class structure*;
- **rank** determines the range of the *power structure* of any society, such that

\[ \text{skills} \times \text{property} \times \text{rank} = \text{Social Structure}. \]

It thus elucidates that identity and difference, equivalence and polar opposites play a crucial role in the constitution of theory than drab notions of tautology and contradiction, or for that matter, Aristotle’s syllogisms suggest.
Ontological and Logical Necessity

At first sight, our imagination seems at a loss with the fourth quadrant. Its truth-values merely duplicate the tabulation of A or B and thus seem of little cognitive value. The only analogue they call to mind is Mendel’s duality of dominant and recessive genes. As another analogue let us imagine a bus line in which A denotes the schedule, and B, the number of passengers or the weather. Then it is possible to say that “whatever the number of passengers, or whatever the weather, the bus will go as scheduled,” i.e., “Under any circumstances A, no matter whether B or not,” or simply, “Necessarily A.” i.e.,

\[ A \land (B \lor \lnot B) = \text{N} A. \]

Likewise, observing a forest fire B, we may wonder whether it was caused by lightning or arson (A or non~A), i.e., "Whether A or non~A, in any case B," i.e.,

\[ (A \lor \lnot A) \land B = \text{N} B. \]

The negative variants to the two examples are easy to imagine. For example, if diabetics must abstain from candy, the rule runs: “Under any circumstances, no candy (~A), whether the patient likes it or not (B or ~B)

\[ \lnot A \land (B \lor \lnot B) = \text{N} \lnot A, \]

or accordingly

\[ (A \lor \lnot A) \land \lnot B = \text{N} \lnot B. \]

All four examples illustrate the dominance of one variable over the other. Accordingly, the truth-value of the function is always that of the argument which dominates, i.e., TTFF and FFTT, respectively, TFTF and FTFT.

In every case the necessity that the statement expresses is judgmental. Necessity and impossibility, possibility and probability are therefore not ontological, but logical categories which imply no causal or teleological determinism. What is commonly categorized as modal logic thus is concerned with modes of judgment and as such replaces – and clarifies – the classical dispute about truth \textit{a priori} and truth \textit{a posteriori}.

Truth-values as Logical Imperatives

The preceding observations highlight the logical imperatives which inhere in all logical functions, including even the weakest one, viz. disjunction. The equation

\[ A \lor B = \lnot A \mid \lnot B = \lnot (A \land \lnot B) = \lnot P (\lnot A \land \lnot B) \]
reveals the hidden imperative which compels us to infer: "Given non-\(\neg A\), necessarily \(B\)",
respectively, "Given non-\(\neg B\), necessarily \(A\)". In every case, the imperative that inheres in
every truth function is brought out by its four truth-values:
\[
A \lor B. = \ \text{P } AB, A\neg B, \neg AB; \neg \text{P } \neg A \& \neg B. = \text{TTTF}.
\]
Likewise, if \(A\) excludes \(B\), \(A\) and \(B\) cannot both be true:
\[
A \mid B. = \neg (A\&B). = \neg \text{P } AB; \text{P } A\neg B, \neg AB, \neg A\neg B. = \text{FTTT}.
\]
The same rule holds for conjunction: If \(A\) and \(B\) form a Cartesian product, neither \(A\)
nor \(B\) can be negative, i.e.,
\[
A \& B. = \neg (\neg A \lor \neg B) = \text{N } AB; \neg \text{P } \neg AB, A\neg B, \neg A\neg B. = \text{TFFF}.
\]
Likewise, if neither \(A\) nor \(B\) are true, any combination of \(A\) and \(B\) is impossible:
\[
A \downarrow B. = \neg A \& \neg B. = \neg \text{P } AB, \neg AB, A\neg B; \text{N } A\neg B. = \text{FFFT}.
\]
In sum, necessity (\(N\)) and impossibility (\(\neg \text{P}\)) are the imperatives which determine the
four truth functions of the first quadrant:

<table>
<thead>
<tr>
<th>Table 7</th>
<th>The First Quadrant in Terms of Modal Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A \lor B</td>
<td>A \mid B</td>
</tr>
<tr>
<td>TTTF</td>
<td>FTTT</td>
</tr>
<tr>
<td>= \neg \text{P } \neg A &amp; \neg B; \text{P } AB, A\neg B, \neg AB;</td>
<td>= \neg \text{P } A &amp; B; \text{P } A\neg B, \neg AB, \neg A\neg B</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>FFFFT</td>
<td>TFFF</td>
</tr>
<tr>
<td>= \text{N } \neg A &amp; \neg B; \neg \text{P } AB, A\neg B, \neg AB</td>
<td>= \text{N } A &amp; B; \neg \text{P } A\neg B, \neg AB, \neg A\neg B</td>
</tr>
</tbody>
</table>

A reduction of the second quadrant to modal form is no less instructive. The asymmetry
of the functions finds its expression in the combination of necessity with non-necessity
as its open-ended counterpart. Thus entailment combines variant antecedents with a
necessary consequent; conversely foundation combines a necessary antecedent with vari-
ant consequent, just as randomness and independence bracket necessity of the consequent
and the antecedent, respectively.
The reduction of the third quadrant to modal form is no less instructive. With unexcelled clarity it brings out the reflexivity of the third quadrant, whose reflexivity allow for no open-ended alternatives.

The Intricacies of Modal Logic
Let us now identify the functions of the fourth quadrant. Like those of the first three quadrants, they complement and explain each other. To say that A is necessary is tantamount to saying that non¬A is impossible, just as saying that non¬A is necessary is tantamount to saying that A is impossible, i.e.,

\[ \neg A \Rightarrow \neg A = \neg A \mid \neg A, \]
\[ \neg A = \neg A \Rightarrow A = A \mid A. \]
By contrast, saying that \( A \) is possible is tantamount to saying that non-\( A \) is not impossible:

\[
P A = \sim N \sim A = P A \lor \sim A.
\]

\[
P \sim A = \sim N A = P \sim A \lor A.
\]

Obviously, contingency (probability and possibility) is the converse to apodicticity (necessity and impossibility). At the same time an unbridgeable gap separates apodicticity from contingency. The first is strict and constitutes logical truth; the second is indeterminate and constitutes factual truth. In sum, modal logic identifies four modes of judgment, which are convertible only to the extent that they recognize the dualism that separates apodicticity from contingency, and *vérités de raison* from *vérités de fait*.

<table>
<thead>
<tr>
<th>Table 10</th>
<th>The Four Modal Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Apodicticity:</strong></td>
<td><strong>Contingency:</strong></td>
</tr>
<tr>
<td><strong>Necessity:</strong></td>
<td><strong>Possibility:</strong></td>
</tr>
<tr>
<td>TTFF</td>
<td>FTFT</td>
</tr>
<tr>
<td>( N A = \sim P \sim A )</td>
<td>( P B = \sim N \sim B )</td>
</tr>
<tr>
<td><strong>Impossibility:</strong></td>
<td><strong>Probability:</strong></td>
</tr>
<tr>
<td>FFTT</td>
<td>TFTF</td>
</tr>
<tr>
<td>( N B = \sim P \sim B )</td>
<td>( P A = \sim N \sim A )</td>
</tr>
</tbody>
</table>

**Quantification and Modal Logic**

A striking homology rules both modal logic and quantification: To say, "all men are mortal" is to say, "there exists not a single man that is not mortal," which is in turn tantamount to saying, "man is necessarily mortal" and "no man is possibly immortal." Obviously, quantifiers and modal logic are homologous.
The gulf that separates apodicticity and contingency thus also separates universal and particular statements (Popper) as well as logical truth and factual truth (Carnap), vérités de raison and vérités de fait (Leibniz). Observation never reaches beyond empirical generalization. It cannot rule out that some x may turn out to be non-Q, i.e.,

$$\exists x \ Qx = \sim \forall x \sim Qx = P \ Qx \lor \sim Qx.$$ 

By the same token factual statements are not convertible into universal statements. Conversely, as Popper (1934) has emphasized, the latter are not amenable to verification.

The term "quantificational logic" thus turns out to be a misnomer: To say, "all x without exception are Q" goes beyond quantitative measurement, just as the statement, "some x are Q" does not specify quantity. What the dichotomy of universal and particular quantifiers really addresses is the distinction between two modes of judgment, apodictic and contingent, or in terms of Leibniz, between rational and factual truths.

Husserl's notion of "existential epoche" addresses the same issue, though from a different angle. To say, "No matter whether centaurs (x) exist or not, they are half man and half horse," is to abstain from factual judgment, i.e.,

$$(\exists x \sim \exists x) \ \forall x \ Qx.$$ 

Conversely, the factual statement, "Socrates is mortal" implies the suspension of universal judgment, or "categorical epoche": "No matter whether all men are mortal or not, this man Socrates is mortal" translates into

$$(\forall x \sim \forall x) \ \exists x \ Qx.$$ 

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We thus end up with the following table, which provides the analytical formula for Husserl’s notion of *epoche: eidetic* reduction bars *phenomenological* reduction and *vice versa* because the two “bracket” each other. As Husserl always emphasized, his phenomenology is based on reduction, i.e., it transcends empirical description.

<table>
<thead>
<tr>
<th>Universal Apodictic Judgments</th>
<th>Particular Contingent Judgments:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q ° ~Q</td>
<td>Q v ~Q</td>
</tr>
<tr>
<td>∀x Qx. = ~∃x ~Qx</td>
<td>∃x Qx. = ~∀x ~Qx</td>
</tr>
<tr>
<td>= (∃x v ~∃x) ∀x Qx</td>
<td>= (∀x v ~∀x) ∃x Qx</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>∀x ~Qx. = ~∃x Qx</td>
<td>∃x ~Qx = ~∀x Qx</td>
</tr>
<tr>
<td>= (∃x v ~∃x) ∀x ~Qx</td>
<td>= (∀x v ~∀x) ∃x ~Qx</td>
</tr>
</tbody>
</table>

The same holds for William James’ “radical empiricism,” which is not an ordinary empiricism. Rather, it is synonymous with *phenomenological* reduction. By contrast, Kant’s notion of transcendentalism turns out to be dual: Looked at more closely, it implies both *eidetic* and *phenomenological* reduction: the first, “making science possible,” the second, founding cognition in the -- radically empirical – “transcendental ego.” It is the conundrum of Kant’s transcendentalism that it addresses both types of reduction without being aware that they represent two different kinds of *a priori*, the first, subjective -- “radically empirical” -- the second, objective and *abstract*. It is the latter which makes science possible, as Kant rightly stated, while wrongly giving credit to the transcendental Ego. The latter cannot possibly explain the objective validity of science, which must be credited to Logic.\(^\text{15}\)

Obviously, the gap between universal and factual statements is unbridgeable. Neither can universal statements be induced on grounds of observation, nor can reality be deduced from universals. Universal statements are "transcendental" in the sense that they transcend the here and now, just as, conversely, “radical empiricism” reduces reality to
strictly existential statements which are embedded in the here and now – the principle which it shares with historicism. It is in this sense that Bergson and William James were true “transcendentalists” in a sense that Kant was not.

The fourth quadrant is therefore fundamental for the philosophy of science. What Carnap addressed as factual versus logical truth juxtaposes unique, contingent action with abstract, ideal sets, an insight to which Wilhelm Windelband (1883) came close with his dualism of nomothetic and idiographic sciences. The same dualism has found its expression in Roland Robertson’s (1974) pregnant distinction between “actionness” and “systemness” as the axes of sociological theory.

<table>
<thead>
<tr>
<th>Table I3</th>
<th>Radical Empiricism and Analytical Science</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROBERTSON:</td>
<td>ACTIONNESS</td>
</tr>
<tr>
<td>Carnap:</td>
<td>contingent</td>
</tr>
<tr>
<td>Leibniz:</td>
<td>“Factual” Truth</td>
</tr>
<tr>
<td>Windelband:</td>
<td>vérités de fait</td>
</tr>
<tr>
<td>Rescher:</td>
<td>idiographic sciences</td>
</tr>
<tr>
<td>W. James:</td>
<td>Correspondence Theory</td>
</tr>
<tr>
<td></td>
<td>metaphysical truth</td>
</tr>
<tr>
<td></td>
<td>existential a priori</td>
</tr>
</tbody>
</table>
Two Types of A priori

The previous table sensitizes us to a fundamental change in our conception of the *a priori*. As Husserl’s duality of *eidetic* and *phenomenological* reduction indicates in tandem with Bergson and William James, the contrast between the two modes of reduction is not that between *a priori* and *a posteriori*, as suggested by Kantianism – including Windelband, Rickert and Max Weber -- but between two kinds of *a priori*, the first, abstract and transcending the here and now; the second, embedded in space and time and thus inherently contingent without being wedded to empiricism, as Husserl’s insistence on phenomenological reduction as opposed to psychology illustrates.

The point is that Husserl’s ingenious notion of *eidetic* reduction is synonymous with structural reduction, which in turn remarkably fits with Occam’s perspicuous notion of *abstract* cognition as opposed to *intuitive* cognition – the first, “transcending” the here and now, the second, embedded in it. It is for this reason that Husserl’s “transcendental” reduction turns out to be a misnomer, for what his phenomenological reduction has to do is exactly reduce empirical reality to something that William James addresses as “radical empiricism” and “metaphysical cognition.” It comes therefore as no surprise that both Husserl and William James sympathized with Dilthey and Bergson. Phenomenological reduction fits historicism once it is understood that the latter implies its own kind of *a priori*, which is no longer wedded to structure and set theory, but to sedimentation in one’s past, both individually and collectively: Everything that exists is rooted in its past, an insight which culminates in Nietzsche’s notion of *amor fati*.

What Husserl called *eidetic* and *phenomenological* reduction thus denotes structural, systematic reduction, respectively, existential, idiographic (biographic) reduction, which replaces abstract systems by concrete, unduplicated Wholes, or in terms of Leibniz, Monads. We have thus to give William James and Bergson credit for having preceded Husserl, we also have to mention the three eminent thinkers that preceded them: Occam (1323), Leibniz (1714), and Schopenhauer (1818). At the same time it elucidates that for all its seeming simplicity, empiricism, including empirical sociology, turns out to be a duplicitous mixture of both kinds of a priori without recognizing them.
As the preceding analyses demonstrate, the sixteen truth functions become fully transparent only if they are arranged in the four quadrants which reveal their specific rationale, viz. commutativity, irreversibility, reflexivity, and recidivity, or reduction. Most important, the widespread conception of a strictly formal, “mathematical” logic (logistics) is complemented by the notion of a substantive logic, which is clothed in language as an instrument not only of communication, but of cognition. What has generally touted as “linguistic turn” thus turns out to be the turn to semiology in the sense of C.S. Peirce and C.I. Lewis (1932) and to "transcendental logic" in the sense of Husserl (1929). The myth of logic as a sort of Platonic side-heaven that miraculously underlies reality thus evaporates together with its positivist counterpart, which reduces logic to a mere technology regardless of cognitive implications.\textsuperscript{16}

As the following tables demonstrate, the sixteen truth functions produce miracles of consistency, variability and perspicuity once they are arranged by quadrants:

<table>
<thead>
<tr>
<th>Table 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>The First Two Quadrants: Mapping Theory</td>
</tr>
<tr>
<td>I.</td>
</tr>
<tr>
<td>A \lor B</td>
</tr>
<tr>
<td>T T</td>
</tr>
<tr>
<td>T F</td>
</tr>
<tr>
<td>A \downarrow B</td>
</tr>
<tr>
<td>F F</td>
</tr>
<tr>
<td>F T</td>
</tr>
</tbody>
</table>

\textsuperscript{16}
Table 15
The Second Two Quadrants: Identity and Modality

<table>
<thead>
<tr>
<th>III.</th>
<th>IV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \neq B$</td>
<td>$A \text{opp } B$</td>
</tr>
<tr>
<td>$TT$</td>
<td>$FT$</td>
</tr>
<tr>
<td>$TT$</td>
<td>$TF$</td>
</tr>
<tr>
<td>$A = B$</td>
<td>$A \leftrightarrow B$</td>
</tr>
<tr>
<td>$FF$</td>
<td>$TF$</td>
</tr>
<tr>
<td>$FF$</td>
<td>$FT$</td>
</tr>
</tbody>
</table>

The two axes which marshal the four quadrants thus reveal the rationale, as it were, the "deep structure" of logic, viz. symmetry vs. asymmetry in the vertical, and description vs. construction in the horizontal axis:

Table 16
The Two Axes of Logic

<table>
<thead>
<tr>
<th>Description (correspondence)</th>
<th>Symmetry:</th>
<th>Asymmetry:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I. commutative</td>
<td>II. unidirectional</td>
</tr>
<tr>
<td>Coordination (atomic sets)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Determination (coherence)</th>
<th>Identification (predication)</th>
<th>Modes of Judgment (quantification)</th>
</tr>
</thead>
<tbody>
<tr>
<td>III. reflexive</td>
<td>IV. reductive</td>
<td></td>
</tr>
</tbody>
</table>

50
Logic Pure and Applied

In contrast to "pure," formal logic applied logic is intrinsically dual and in this sense, synthetic. Its rigorous character notwithstanding, it combines formal calculi with substantive arguments. It is therefore subject to falsification. By the same token, logical truth (L-truth) is subject to continual revision both of its variables and its functions. The first focuses on piecemeal revision (Popper 1934); the second, on paradigm change (Kuhn 1970).

Kant's notion of synthesis a priori thus finds a new interpretation: It is the synthesis of substantive content with logical grammar, i.e., of substantive reference (Bedeutung) with syntactical meaning (Sinn). Taken by itself, logic would be confined to the ivory tower.

<table>
<thead>
<tr>
<th>pure logic:</th>
<th>applied logic:</th>
</tr>
</thead>
<tbody>
<tr>
<td>p v q</td>
<td>denotes adjacency, inventory, enumeration</td>
</tr>
<tr>
<td>p ↓ q</td>
<td>denotes joint denial, rejection, dismissal</td>
</tr>
<tr>
<td>p</td>
<td>q</td>
</tr>
<tr>
<td>p &amp; q</td>
<td>denotes complementarity, inseparability, intersection</td>
</tr>
<tr>
<td>p → q</td>
<td>denotes sufficient reason, necessary consequence, causality</td>
</tr>
<tr>
<td>p acc q</td>
<td>denotes randomness, accidentalness, chance, openness</td>
</tr>
<tr>
<td>p ← q</td>
<td>denotes necessary condition, foundation, emergent hierarchy</td>
</tr>
<tr>
<td>p ind q</td>
<td>denotes discontinuity, rupture, leaps, fragmentation</td>
</tr>
<tr>
<td>p ≠ q</td>
<td>denotes uniqueness, singularity, difference</td>
</tr>
<tr>
<td>p = q</td>
<td>denotes contradiction, invalidity, error</td>
</tr>
<tr>
<td>p ↔ q</td>
<td>denotes equivalence, synonyms, substitution</td>
</tr>
<tr>
<td>p opp q</td>
<td>denotes contravalence, antonyms, polar opposites</td>
</tr>
</tbody>
</table>

| N p | = ~P ~p; | denotes necessity, apodicticity, certitude |
| N ~p | = ~P p; | denotes impossibility, elimination of error |
| P q  | = ~N ~q; | denotes possibility, novelty, creativity |
| P ~q | = ~N q; | denotes probability, conjecture, measurement |
We thus end up with a clear distinction between pure and applied logic: "Pure," algebraic logic is confined to truth-possibilities (*Wahrheitsmöglichkeiten*) and truth-conditions (*Wahrheitsgründe*). These turn into truth-values when saturated variables are substituted for the unsaturated ones. By the same token, formal logic is intrinsically analytic, just as applied logic, i.e., language, is intrinsically synthetic. In any case the conjunctional normal form reveals the truth-conditions for each of the 16 truth-functions. The following table demonstrates the unity of logic and by the same token, its completeness and self-sufficiency:

<table>
<thead>
<tr>
<th>Matrix:</th>
<th>Conjunctural Normal Form:</th>
<th>Truth-Conditions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTTF</td>
<td>pvq</td>
<td>P pq, <del>pq, p</del>q; ~P <del>p</del>q</td>
</tr>
<tr>
<td>FFFT</td>
<td>p↓q</td>
<td>~p &amp; ~q</td>
</tr>
<tr>
<td>FTTT</td>
<td>p↑q</td>
<td>~p &amp; q</td>
</tr>
<tr>
<td>TFTF</td>
<td>p&amp;q</td>
<td>~p &amp; q</td>
</tr>
<tr>
<td>TFFT</td>
<td>p→q</td>
<td>~p &amp; q</td>
</tr>
<tr>
<td>TTTT</td>
<td>p≠q</td>
<td>p&amp;p&amp;.q&amp;q&amp;.q</td>
</tr>
<tr>
<td>FFFF</td>
<td>p=q</td>
<td>N p=q</td>
</tr>
<tr>
<td>TFFT</td>
<td>p↔q</td>
<td>(p v ~p) &amp; q</td>
</tr>
<tr>
<td>FTFF</td>
<td>N p</td>
<td>p &amp; (q v ~q)</td>
</tr>
<tr>
<td>FFFT</td>
<td>N ~p</td>
<td>~p &amp; (q v ~q)</td>
</tr>
<tr>
<td>TFTF</td>
<td>N q</td>
<td>(p v ~p) &amp; q</td>
</tr>
<tr>
<td>FTFT</td>
<td>N ~q</td>
<td>(p v ~p) &amp; ~q</td>
</tr>
</tbody>
</table>

It is therefore erroneous to equate strict science with mathematics or with formalization. In every case, it is logic that produces apodictic truth, which is not lim-
ited to the natural sciences. The point is that each of the sixteen truth-functions can be converted into any other salva veritate, i.e., so long as its truth-values remain unaltered, as De Morgan's Law anticipated:

\[ A \& B. = \sim(\sim A \lor \sim B). = \sim A \downarrow \sim B. = FFFT. \]

Clearly, human thought is predicated on logic, to which it owes its rigor and precision even at a pre-linguistic stage.

**Logic as a Whole**

Contrary to the belief that logic is strictly technical and bare of quality, applied logic reveals the specific qualities of each of the four quadrants and of each of the sixteen truth functions. Just as truth is not separable from meaning, applied logic is synonymous with cognition.

![The Four Quadrants of Applied Logic](image)
THE LOGICAL FOUNDATIONS OF THEORY

The Dual Roots of Logic and Pragmatism

Our emphasis that all necessity and possibility is logical rather than ontological does not prevent us from asking about the ontological foundations of logic. The answer to this question is as surprising as it is instructive: Just as Husserl distinguished between two modes of reduction, our knowledge stems from two quite different sources: *eidetic*, or structural reduction, which is analytic, and *transcendental* or *phenomenological* reduction, which is synthetic. The first reduces the perceived reality – not reality *per se* – to homogeneous, but abstract sets. By contrast, *transcendental* reduction synthesizes (unifies) the Person’s experiences into a meaningful whole, which constitutes his existence. The latter thus represents a work of art of the Person’s own making which is in constant flux because it changes with every subsequent decision.

Husserl is thus attending to a type of psychology which radically changes Kant’s notion of transcendentalism. Like Fichte and Schelling, Schopenhauer and Nietzsche, Husserl no longer asks how (objective) science is possible, but rather, how the “pure” (transcendental) Ego works when it is unencumbered by external constraint. Husserl “pure” Ego works like a Leibnizian *Monad*. He thus ends up with (subjective, real, ever changing) *Noeseis* and (objective, ideal, abstract) *Noemata* as the primal elements of cognition and of action.

Husserl thus deserves credit for opening a new approach to philosophy which is in striking parallel with Bergson and William James, with the important difference that Husserl remained focused on logic, while James focused on Praxis as the primal givens which for him, like for Bergson, overlapped with Ethics and Religion. One has to widen the picture to include German historicism and French existentialism to obtain the full picture of James’ “radical empiricism.” The latter starts a philosophy which is no longer focused on abstract *noemata* – and that is, on logic -- but on real, spontaneous, contingent acts which eschew logic, being predicated on Schopenhauerian elementary Will, Bergsonian *élan vital* and Nietzschean “Will to Power,” with Leibnizian *Monads* as the famed, if fictitious precursors.
What was not clear to Kant and Fichte is the fundamental difference between impersonal sets and truth-functions, on the one side, and contingent, spontaneous acts, on the other side: The first transcending space and time; the second irrevocably embedded in time and space, and even where they anticipate the future or memorize the past enchained with the present.

Whether he deals with Perikles or Ceasar, Benjamin Franklin or Thomas Paine, the historian describes their life in the present tense. For the same reason the existentialist describes action as churning the future out of the present. History and existentialism thus sensitize us to the fact that action generates time.

By the same token action resists generalization. The latter is the hallmark of logic and of science. By contrast, as epics, the drama and the novel illustrate, action is predicated on biography. Every biography thus constitutes a self-sufficient Monad which follows its own rationale, which James identifies as “radical empiricism,” and which is identical with his notion of pragmatism.

There exists therefore a vast area which is different from logic and is not reducible to it, viz. action, which follows its own rationale: the never-ending, unrelenting quest for subjective meaning, or meaningfulness. The latter permeates the work of William James, who at the same time expressly recognizes abstract concepts as indispensable instruments to cope with the external environment. Logic is therefore ubiquitous in all our dealings with our surroundings, starting with the first flint used as a knife and the first cries to express joy or to warn of danger, with artifacts as its elementary instruments.

Husserl’s two modes of reduction thus provide the keys to reality: to analyze and to explain the external world, on the one hand, and to ”understand” and empathize action, on the other hand; the first, culminating in logic and science, the second, culminating in pragmatism, ethics and religion as the Sinngebung des Sinnlosen.

What Husserl disguised as eidetic and transcendental or phenomenological reduction thus turns out to address structural, respectively, existential reduction, the first known since Plato and Pythagoras, the second anticipated by Leibniz, but ultimately secured only by Dilthey and Husserl, Bergson, William James and Sartre under so many different names.
Logic, Language and Mathematics

Much of the pioneering work of modern logic has focused on pure (mathematical, algebraic, formal) logic, including advances in modal logic and set theory, but relatively few efforts have been made to draw a sharp distinction between pure and applied logic and to create a distinct theory of pragmatic, effective logic in terms of Peirce and Lorenzen. The result has been a tendency to consider logic an esoteric discipline like higher mathematics or to reduce it to mere convention in the guise of language games.

It seems therefore timely to supplement formal logic with a substantive, cognitive interpretation which reaches beyond mere tautologies. Like tools and signs, whose rationale lies in their use, the functions of logic must be seen as cognitive instruments which serve to come to grips with reality. As Lorenzen and the intuitionists (e.g., Brouwer 1983) have argued, there exists a primitive, inchoate logic of the senses, a sort of protologic, which Quine (1995, p.23), too, recognizes in birds, apes and infants, and which may be the reason for his naturalistic interpretation of logic.

In any case, a sharp distinction must be made between language, logic and mathematics. While they are all three products of the intellect, they serve different functions: Language is man's most eminent instrument of communication -- and as such, the well-spring of meaning -- and mathematics, his most eminent instrument of measurement and computation. By contrast, logic is man's most eminent instrument of thought, which lies at the bottom of both language and mathematics, as it were, as its deep structure.

While language creates icons, indices and symbols as the necessary carriers of meaning (Peirce 1955), what lies at the heart of logic is the combination of four truth-values, or more precisely, truth-possibilities -- Wittgenstein's (TLP 5.105) ingenious discovery. The latter underlie all thought no matter what its content and thus supports the idea of Einheitswissenschaft (Schlick 1928) without reducing it to physics or mathematics. If anything establishes the unity of science it is logic.

The Constitution of Theory

To elucidate the cognitive function of logic, one problem must be solved in the first place: How reality translates into logic. The task is as much to elucidate the logical foun-
dations of cognition as to elucidate the cognitive foundations of logic. Both complement each other. This goal is achieved in three steps:

(1) To overcome the plethora of empirical observations, all particular, factual statements must be reduced to universal, apodictic statements. In contrast to ordinary, everyday language, which is based on indexicality, logical truths are the product of abstraction and generalization, which transcends the here and now. Logical truth is therefore radically different from factual truth.18

(2) The next step is to reduce our language to simple “atomic sentences,” or "canonical notation" (Quine 1960: 226ff), i.e., to elementary propositions, which are stripped down to two variables and whose relationship is regimented by one of the sixteen truth functions. The latter constitute the syntactic meaning, or the logical syntax (Carnap), of human thought.

(3) The two preceding steps provide the building blocks for analytics. However, to create theory one more step is required which is missing in most textbooks: systematicity, or in Kantian terms, architectonics. As Kant (1781, A 474) put it:

Human reason is by nature architectonic. That is to say it regards all our knowledge as belonging to a possible system and therefore allows only such principles as do not at any rate make it impossible for any knowledge that we attain to combine into a system with other knowledge.

In a special chapter on "The Architectonic of Pure Reason" Kant (1781, A 832) elaborates:

By an architectonic I understand the art of constructing systems. As systematic unity is what first raises ordinary knowledge to the rank of science, that is, makes a system out of a mere aggregate of knowledge; architectonic is the doctrine of the scientific in our knowledge...

In accordance with reason's legislative prescriptions our diverse modes of knowledge must not be permitted to be a mere rhapsody, but must form a system...

It is the unity of the system which makes it possible for us to determine from our knowledge of the other parts whether any part be missing, or to prevent any arbitrary addition...
The whole is thus an organized unity and not an aggregate. It may grow from within, but not by external addition. It is thus like an animal body the growth of which is not by addition of a new member, but by rendering each member... stronger and more effective for its purposes.

While we do not share Kant's belief in a priori knowledge independent of experience, we owe him the cue for defining strict science: systematicity. The latter coincides with his notion of pure reason (Vernunft) as the source of apodictic truth, which in turn fits with the dichotomy of cognition ex datis and cognition ex principiis (A 836).

While it makes sense to interpret Kant's notion of pure reason as pure, formal logic, the latter fails to establish any substantive link with reality. Ironically, the solution of the puzzle lies in Kant's notion of architectonics. In his own terms, systems are wholes whose parts are intricately connected and explain the whole as much as the whole explains the parts. Theory is therefore not founded in observed regularities, but in the validity of its architectonics. Its stringency notwithstanding, it allows for, and indeed invites, free Ideenvariation so long as the truth-values remain unaltered.

It is thus the necessary connexion of forms -- Plato's notion of sympleke ton eidon -- that constitutes rational truth. What starts with the infants' perception of identities and differences thus culminates in Aristotle's hierarchy of matter, plants, animals and humans, in Newton's law of gravity, in Planck's quantum theory and in Einstein's theory of relativity. They all derive the validity of their theses not from observed regularities, but from the compound of logical consistency, parsimony and completeness that lies at the bottom of Kant’s notion of architectonics.

**Ontological Relativity and Logical Determinacy**

The notion of logical regimentation also sheds light on "ontological relativity" (Quine 1969). As Althusser (1968), following Lenin, has argued, historical reality is overdetermined in the sense that real (natural and historical) events are determined by a plurality of contingent factors. By contrast, logical regimentation produces ideal constructs which, for the sake of Denkökonomie (Mach) and simplicity (Bunge), bracket all intervening factors. Inevitably, parsimony and precision are bought at the cost of completeness.
Conversely, ontological completeness is bought at the price of analytical determinacy. Looked at more closely ideal constructs and "things-in-themselves," analysis and intuition are polar opposites which are mutually exclusive: Logical determinacy implies ontological relativity; conversely, observational plenitude implies logical indeterminacy. Which truth function to choose is for the observer to decide. In the last analysis, "ontological relativity" is a misnomer which attributes the indeterminacy of reference to the reality it refers to (Quine 1990: 50).

For an illustration, let us consider a situation which is characterized by the interaction of two variables, a and b, in an indeterminate setting K, e.g.,

\[(a + b) \times (a + b) + K = aa + 2ab + bb + K.\]

Then differentiation toward a \((\partial a)\) results in

\[\partial a (aa + 2ab + bb + K) = 2a + 2b,\]

i.e., all combinations not containing "a" are bracketed from our view. For all its precision the differential is underdetermined compared with the plenitude of the original equation. This becomes all the more clear if we decide to restore the original plenitude by way of integral calculus:

\[\int a (2a + 2b) = aa + 2ab + b + c + .... + K....\]

allows for an indefinite variety of additional variables (c, d, etc.) and constants (K), leaving everything undetermined that is not related to a. Invariably, specific determinacy is bought at the price of general indeterminacy, and vice versa, i.e., analyticity is inversely related to indexicality. The notion of ontological pluralism thus sensitizes us to the fact that the choice of truth functions is in the eyes of the observer rather than in the "thing itself." It is the fundamental fallacy of realism to believe otherwise.

**Logic and Epistemology**

A glance at the strict tautologies of formal logic highlights the difference between pure and applied, formal and, for lack of a better term, "material," or in terms of Husserl (1929), "transcendental" logic. It is in this sense that logic is dual. On the one hand, applied logic owes its precision, validity and objectivity to formal logic. On the other hand, applied logic establishes the link between monothetic form and polythetic perception, and that is, between syntax and semantics.
For all the rigor it owes to "logistics," science gains immensely in perspicuity, scope and versatility by combining formal rigor with perception, as the leap from strict implication to causality and foundation, randomness and independence and from universality and particularity to necessity and impossibility, possibility and probability demonstrates.

By the same token strict science is no longer limited to mathematics and the natural sciences. Applied logic covers anything from physics to society, ethics and culture. In sum, the rift between natural and "human sciences" is bridged as soon as the right functions combine with the right variables. Accordingly, the way to sociology as a strict, analytical science leads neither through mathematics nor through formalization, but through set theory.

June 26, 2003
Endnotes

1By the same token, formal logic transmutes into transcendental logic.

2Quine admits: "No doubt, this result seems strange," but adds that it would not be any less strange to construe it as false.

3Patrick Suppes (1957: 15) and W.V. Quine (1983) address the same point when they speak of "tautological" and "logical" implication, respectively.

4Sartre's work on Being and Nothingness in tandem with Bachelard's notion of rupture and discontinuity may be considered the philosophical equivalent to the theories of Einstein and Planck.

5Neither Wittgenstein (TLP 5.101) nor Bochenski (1959) nor Carnap (1954) or Quine (1982; 1986) developed the notion of quadrants. On the other hand, the notion of converse, joint denial and dual implies the notion both of an elementary zero-form and of a quadrant, with De Morgan's (1842) law as the earliest manifestation. A notable exception is Paul Lorenzen (1965: 34f), who addresses the 1st and 2nd quadrants.

6Thus Quine (1990: 14) divides logic into truth functions, quantification, and identity. In a similar fashion Lorenzen (1965) distinguishes between logic of functors, logic of quantors, and logic of identity.

7We thus take the Fregian and Russelian position that mathematics presupposes logic. While we often speak of formal logic as "mathematical" logic insofar as it remains limited to pure schemata, what we really mean is "algebraic" logic. In no case is logic reducible to mathematics.

8By the same token the third quadrant has nothing to do with quantification, which constitutes the fourth quadrant. Indeed Lorenzen (1965: 75) criticizes Quine for subsuming the logic of quantors under predicate logic. He also observes that predicate logic is often subdivided into lower and higher predicate logic, which correspond to quantification and set theory, respectively.

9It is noteworthy that Wittgestein (TLP 5.101) did not commit this error. He interpreted tautology as "p→p.&.q→q," and contradiction as "p&~p.&.q&~q".

10In terms of arithmetic, identity is defined as 1x1 = 1, and 1:1 = 1.

11One is reminded of Aristotle's formula Omnis definitio fit per genus proximum et differentiam specificam.

12It is thus obvious that the so-called linguistic turn, if it is to be taken seriously, is really a logical turn.
For an application of this idea to social structure see Sorokin 1964 and Mueller 1989 on Analytical Sociology.

Note that Wittgenstein's (1922, p. 75) computation of the 16 truth functions does not recognize the four modal functions as such. The same holds for Bochenski and Menne (1962, p. 31), who speak of "Präpendenz," "Postpendenz," and their denials, "Pränonpendenz," "Postnonpendenz."

In other words, Kant never realized that his transcendental philosophy was limited to *eidetic* reduction and did not really deal with the "transcendental Ego."

The attacks on logistics, e.g., by Freytag-Löringhoff (1961) centered on the failure to link logic with cognition and language, a shortcoming which the "linguistic turn" tried to mend at the expense of formal logic.

Peirce's (1955, ch. 7) trichotomy remarkably parallels Freud's trichotomy of Id, Ego and Superego. It recognizes protologic in the form of icons and thus concurs with Lévi-Strauss' (1968) notion of *The Savage Mind*.

In terms of Husserl (1900) it is the function of logic to transform polythetic perceptions into monothetic truth.

Kant thus attributed to an imaginary "pure reason" what rightly belongs to logic. What Kant could not know is that his *syntheses apriori* reduce to the sixteen truth functions.

Accordingly Kant (1781, A 371) considers himself an empirical realist as well as a transcendental idealist depending on which kind of cognition he refers to.

For the same reason Hegel has a point when he notes that all our proofs are circular.

By the same token Hegel had a point when he noted that all our proofs are ultimately circular.
III.

ANALYTICITY AND TOTALITY

Mapping Functions and Mappings
The distinction between “pure,” algebraic logic and applied, everyday logic has sensitized us to the difference between “unsaturated” and “saturated” statements. Only the latter produce true or untrue sentences. This holds in particular for the simplest form of cognition, as it were, “atomic sentences:” While (x) is an undetermined algebraic variable, its identification with concrete, real objects produces meaning such as “(x) is a dog,” which holds not only for one dog here and now, but for all (x) that are dogs.1

By the same token a dog (x=A) is not a cat (x≠B), i.e.,

\[ A \neq B = \neg(A=B) = A=\neg A, B=\neg B = TTTT, \]

whereas \( A=B \) denotes contradiction, i.e.,

\[ A = B = \neg(A \neq B) = A=\neg A, B=\neg B = FFFF. \]

In other words, \( A \neq B \) denotes both difference and identity, i.e.,

\[ A \neq B = \neg(A=B) = A=A, \neg A=\neg A, B=B, \neg B=\neg B. \]

At the same time, the statement “Sultan (x) and Caesar (y) are dogs” may be read in the inverse direction. Looking for dogs among the animals in our neighborhood, we may end up with Sultan and Caesar, i.e., in terms of mapping theory:

\[ \text{dogs : } \{\text{Animals}\} \rightarrow \text{Sultan, Caesar,} \]

i.e., from the empirical domain of \( \{\text{Animals in our neighborhood}\} \) the mapping function \textit{dogs} selects Sultan and Caesar. In other words, \textit{mapping functions} (underlined) relate to their Mappings (Capital initial) like intensional meaning to extensional Mapping.

While there is no doubt that reality exists independent of cognition, it is no less true that there are no Mappings without mapping functions. An asymmetrical relationship thus exists between mapping functions, which are based on identity, and Mappings, which are contingent: On the one hand, inadequate Mappings call for the revision of the functions, which in spite of their ideal, “transcendental” character are not immutable. On the other hand, once a mapping function is changed, the new meaning alters the Mappings, i.e., the perceived (empirical) reality.
Clearly the relationship between meaning and Mapping, ideal intension and contingent extension is asymmetrical. The same holds for the relationship between Language and Speech, monothetic meaning and polythetic cognition. Theory is therefore subject to change if its mapping functions are out of tune with perceived facts. Yet in contrast to piecemeal change, which contents itself with cumulating facts, change in a mapping function implies “paradigm change” throughout the text. Accordingly, Thomas Kuhn’s thesis is asymmetrical. It effectively undercuts the belief in unalterable categories, but stays silent about the duality of mapping functions a priori and Mappings a posteriori.

Wittgenstein’s Theory of Language Games
The preceding analysis provides the key to the structure of language. While it is certainly based on simple ostension and social convention – much in the vein described by G. H. Mead – the point is that language relates to cognition like mapping functions relate to Mappings. Looked at more closely, cognition implies the creation of signs as material carriers of meaning. Semiology lies at the bottom of epistemology.

It thus seems, as Wittgenstein has argued, that cognition depends on the accurate use of language – a supposition which is fundamentally flawed. The point is, as Wittgenstein never tired to emphasize, that language is based on convention and is hence inherently contingent. Whatever its logical inadequacies – about which Wittgenstein stays silent – language works so long as it serves communication, no matter whether the whale is a fish or whether the sun rotates around the earth.

Accordingly systematicity – i.e., consistency, completeness and parsimony – yields to “grammar” and to “following a rule” – two of Wittgenstein’s favorite arguments. The entire “linguistic turn” that was triggered by Wittgenstein (1953) is therefore deliberately bared of logic and the stringency of self-explanatory truth-functions. The erstwhile logician Wittgenstein thus prides himself in deconstructing theory, implying that “following a rule,” “grammar,” and convention suffice to eradicate error.

The irony is that for all his railing against theory Wittgenstein has indeed made a fundamental discovery about language: It combines the asymmetry of meaning and Mapping, intension and extension – as it were, walking in Kant’s footsteps – with a different kind of a priori that is no longer objective and transcendental but situational or existential.
in the sense that it is inherently here and now and in spite of its contingency, determinate:

In terms of Sartre, there are no projects that determine our future without realizing the
given situation that limits possible projects without determining them, i.e.,

\[ \text{SITUATION} \leftrightarrow \text{PROJECT.} = \sim \text{SIT} \Rightarrow \sim \text{PROJ.} = \text{PROJ} \rightarrow \text{SIT.} \]

What has been called the “linguistic turn” thus marks the discovery of a second type of a
priori that lies at the bottom of existentialism and historicism, but in no way replaces the
structural “transcendental” a priori that in terms of Kant “makes science possible.”

The picture changes fundamentally once instead of mapping Sultan and Caesar as the
empirical extensions of dogs we ask how we arrive at the notion of cat and dog, i.e. in
terms of mapping theory

? : [Animals] \rightarrow \text{Sultan, Ceasar}

The answer is that we have to find out the distinctive qualities which distinguish
dogs from all other animals. It is therefore not enough to define dogs as mammals that
suckle their young and walk on four legs (which distinguishes mammals from fish and fowl). Rather, we have to find out what it is that makes dogs differ from all other mammals, viz. having snouts to attack and defend themSelves and being able to bark. In other
words, Mappings represent Sets, which are defined by their distinctive qualities. Set the-
ory also facilitates the decisive step back from pure, algebraic logic to substantive, practi-
cal logic that is embedded in every day speech.

Some new light also falls on de Saussure’s (1916) distinction between langue and pa-
role, viz. the difference between ideal structure and contingent, real Speech, which has
become the founding document of modern structuralism with Lévi-Strauss and Piaget as
its most prominent proponents. It is in this sense that structuralism sheds a new, corrobo-
rating light on classical distinctions such as Wesen and Erscheinung, noumenon and phe-
nomenon, monothetic meaning and polythetic Mapping, all of which find their explana-
tion in set theory. Contrary to the assumptions of empiricism, cognition is inherently
asymmetrical, with ideality (meaning) and contingency (Mapping) as the two poles which
are inseparable but also mutually irreducible.
Polar Opposites, Parameters and Sets

Next to identity and difference the power of logic in its most elementary form is strikingly illustrated by pairs of polar opposites such as hot and cold, young and old, etc. In each case the denial of the first defines the second, and vice versa, i.e.,

\[
\begin{align*}
\text{hot versus cold} & \quad = \quad \text{hot} = \neg \text{cold} \quad \& \quad \text{cold} = \neg \text{hot}. \\
\text{a opp b.} & \quad = \quad a \leftrightarrow \neg b. \quad \& \quad \neg a \leftrightarrow b.
\end{align*}
\]

As Piaget (1932) and Lévi-Strauss (1962) have emphasized, polar opposites lie at the bottom of all Praxis, Ethics and Thought. They also provide the key to analytical theory. The point is that opposites encompass parameters as their common denominator. Thus

- Young vs. old encompasses age
- Male vs. female encompasses gender
- Patrilineal vs. matrilineal lineage encompasses lineage.

At the same time, age, gender and lineage serve as mapping functions which map complex, heterogeneous reality into homogeneous Sets that reduce reality to distinctive qualities and make opaque reality transparent:

\[
\begin{align*}
\text{age} : \text{[People]} & \quad \rightarrow \quad \text{young or old people} \\
\text{gender} : \text{[People]} & \quad \rightarrow \quad \text{male or female people} \\
\text{lineage} : \text{[People]} & \quad \rightarrow \quad \text{patrilineal or matrilineal families}.
\end{align*}
\]

At the same time we note that the three mapping functions constitute a Cartesian product none of whose elements can decline to zero and as such define Kinship, i.e.,

\[
\text{Kinship} = \text{age} \times \text{gender} \times \text{lineage}.
\]

What appears to us as Kinship as the primordial system of social association is thus reduced to its analytical structure. Contrary to primitive assumptions that equate society with molecules which are composed of elements, our method is strictly qualitative, based as it is on distinctive qualities that determine Sets rather than on Wholes and parts that are accessible to simple observation, but which wield only individuals as its components.

Social Indicators, Structure, and Totalities

Kinship Systems such as Clans (gentes), Families and Neighborhoods thus represent empirically verifiable wholes that are primordial forms of association from which Social Structure emerges, not by increasing numbers, but as people develop skills, property and leadership. In terms of primitive opposites and qualitative parameters
skilled vs. unskilled encompasses occupation,
rich vs. poor encompasses property,
superordination vs. subordination encompasses rank.

By the same token skills, property and rank serve as mapping functions that reduce complex, opaque Society to its structure:

rank reduces Society to its power structure;
property reduces Society to its economic or class structure;
skills reduce Society to its occupational structure.

Once Society is reduced to its structure, it is easy to identify the contribution each structure makes to Society:

The occupational Structure promotes production.
The property Structure promotes investment.
The power Structure promotes the organization of Society.

It thus elucidates that our ability to analyze Society is ultimately founded in pairs of polar opposites. Far from being rudiments of primitive realism, polar opposites bring theory down to earth. As Piaget (1930) has shown, human Praxis and Thought are predicated on polar opposites even at a preverbal stage. Whatever the preoccupation of empirical research with quantitative measurement, the roots of theory are strictly qualitative.2 Invariably, quality precedes quantity.

The reduction to structure goes with another fundamental insight: What is commonly addressed as Social Structure constitutes a Cartesian product of skills, property and rank (i.e., production, investment and leadership), none of which can decline to zero, i.e.,

Social Structure = occupational x economic x power structure.

It is in this sense that social reality, like physical reality, is three-dimensional, and while the three dimensions are inseparable, each dimension varies in its own right. Gerhard Lenski has therefore introduced the notion of status inconsistency, which sheds considerable light on social dynamics. For example, high occupational status may combine with low economic and/or political status and vice versa. Lenski thus provides the analytical formula for Pareto’s “foxes” and “lions,” the foxes being low in power, and the lions, high in power, while economic status may go either way, as the Whigs illustrate.

Moreover, a sharp distinction must be made between structures and sets, on the one hand, and concrete Wholes such as Households, Civil Society and the State, on the other hand. The latter are self-regulating totalities that are embedded in individual action while they are coterminous with the structural parameter that determines them. Thus
Households are determined by, and coextensive with, the occupational structure; Civil Society is determined by and coextensive with the economic structure; The State is determined by and coextensive with the power structure of Society.

We thus end up with three different formulas for Society dependent on whether we deal with empirical indicators, abstract parameters, or self-regulating totalities, viz.

- empirical indicators of status = skills x property x rank;
- structural parameters = occupational & economic & power structure;
- complex self-regulating totalities = Households v Civil Society v the State.

Each of the three formulas addresses a different aspect of social theory:

1) skills, property and rank are indicators of social status which are directly observable;
2) abstract parameters reveal the analytical structure of Society;
3) Households, Civil Society and the State constitute self-regulating Wholes, or in terms of Luhmann, "autopoietic systems,” which are embodied in individuals whose actions account for their apparent teleology.

**Personal Ethos and Social Consciousness**

We have thus ended up with an impressively precise analytical picture of social structure which is marked by social status and is hence identical with social stratification. The picture of Social Consciousness, or the Mind, is this far much less unequivocal, which is aggravated by the fact that it is the subject not only of sociology, but likewise of Ethics (Max Scheler and Nicolai Hartmann), of psychology (Freud, C.G. Jung), of cultural anthropology, or *Kulturwissenschaft* (Max and Alfred Weber, Rickert, Cassirer, Michael Landmann, Robert Wuthnow) and of hermeneutics. (Schleiermacher, Dilthey, Paul Ricoeur, Gadamer).

To shorten our discussion, let as assume that the Mind is homologous with Social Structure, while it is fundamentally different in substance: Rather than on social status the Mind is predicated on personal values, which constitute the personal Ethos as its core. To begin with, let us look for polar opposites and the structural parameters that bring the Mind down to earth:

- friend vs. foe encompasses one’s *Ingroup Feeling*;
- consent vs. dissent encompasses one’s *Definition of the Situation*;
- right vs. wrong encompasses one’s *Sense of Justice*.
We thus end up with three parameters which are solidly founded in everyone’s daily experience and which corrects almost all existing analyses of the Mind and its core, the personal Ethos. Rather than on Ideology and false consciousness or Morals the Mind is defined as the Cartesian product of Ingroup Feeling, which is borrowed from Graham Sumner’s *Folkways*, the “Definition of the Situation,” which is borrowed from W.I. Thomas, and the sense of Justice, which goes back to Plato and Kant. Moreover each parameter serves a distinct function:

- a person’s **Ingroup Feeling** determines his or her **belonging**;
- a person’s **Definition of the Situation** determines his or her **social orientation**;
- a person’s **Sense of Justice** determines his or hers **moral judgment**.

We thus end up with a definition of the personal Ethos, or the character structure of the human Personality, which constitutes a Cartesian product, viz.

\[
\text{The personal Ethos} = \text{belonging} \times \text{orientation} \times \text{judgment}.
\]

Graham Sumner’s notion of “Ingroup Feeling” thus replaces Durkheim’s notion of Solidarity, while W. Isaac Thomas’ “Definition of the Situation” replaces the Marxist notion of Ideology and Robert Merton’s notion of Reference Groups, while the “Sense of Justice” revives the classical notion of Morality. By contrast, Historical or Common Consciousness, Status Consciousness and Moral Consciousness denote self-regulating auto-poitetic Systems in the sense of Luhmann, each of which is three-dimensional and waxes or wanes as individuals pursue projects which aim to shape reality. However strict the analytical formulas, the shape of each Totality is contingent on individual decisions and as such is in constant flux:

- Common Consciousness is dominated by, and coextensive with Ingroup Feeling.
- Status Consciousness is dominated by, and coextensive with Definition of the Situation.
- Moral Consciousness is dominated by, and coextensive with the Sense of Justice.

As in the case of Social Structure we end up with three corresponding formulas that highlight various aspects of the Mind, viz. empirical indicators, structural parameters that form a Cartesian product, and contingent, ever fluctuant TOTALITIES that exist one along the others:

\[
\begin{align*}
\text{personal indicators} &= \text{Ingroup Feeling} \lor \text{Definition of the Situation} \lor \text{Sense of Justice}; \\
\text{structural parameters} &= \text{belonging} \& \text{orientation} \& \text{moral judgment};
\end{align*}
\]
contingent TOTALITIES = COMMON CONSCIOUSNESS v STATUS CONSCIOUSNESS v MORAL CONSCIOUSNESS.

Analyticity versus Totality

The suggested triple formulas for Society and the Mind open a new perspective on socio-logical theory which surprisingly parallels Hegel’s triads of primordial, perceived truth -- as it were, “truth by acquaintance” -- followed by “abstract” analytical truth, i.e., reduction to structure, to end up with “philosophical” truth, i.e., the interpretation of reality as an aggregate of Totalities that represent self-regulating wholes which combine abstract structure with the ever fluctuant will of its constituents.

Sartre’s two types of biographic micro Totalities and collective macro Totalities thus represent wholes, or in terms of Leibniz, monads, of quite distinct composition. On the one hand, individual biographies, which represent individual choices which determine their fate; on the other hand, “collective representations” with no suprapersonal will and no other unity than some “family affinity” among each individual’s notion of “ultimate meaning,” which represent religion and philosophy, whose sum is the contingent outcome of their interaction, starting with simple individual PRAXIS which leaves the individual monad free of structural constraints, and culminating in the Social Consciousness as the contingent outcome of interacting personal monads.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Property</th>
<th>Skills</th>
<th>Occupational Structure</th>
<th>Property Structure</th>
<th>Leadership</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDIVIDUAL PRAXIS</td>
<td>SELF-REGULATING HOUSEHOLDS</td>
<td>CIVIL SOCIETY</td>
<td>THE STATE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this way self-regulating Households emerge from simple Kinship relations by developing an occupational structure that survives the individual participants while property and leadership continue to emerge and vanish freely until property develops a consistent economic structure which survives the individual participants and relegates the occupational structure to subsidiarity while leadership continues to wax and wane.
freely. Finally, when leadership develops into a compact **power structure**, Civil Society yields to the State.

Structure thus reveals the hard core (as it were, the essence) of each of the three subsystems of Society: the State is coextensive with, and determined by, its power structure, which amalgamates with the economic and the occupational structure. Private property is thus complemented with public property, taxes, dues and tariffs, just as self-employed labor and wage labor is complemented with salaried officers, officials, jurists and teachers whose jobs are independent of Civil Society and Family bonds.

In a similar fashion Civil Society reduces labor to a subsidiary and thus complements self-employed labor with wage labor. At the same time Civil Society opposes being converted into a subsidiary of a dominant power structure. What is commonly celebrated as division of powers thus reveals the essence of Civil Society, which is equally opposed to state power and to the redistribution of property.

By contrast, socialist and populist movements are opposed to state power and to the dominance of the bourgeoisie. Socialism and anarchism are thus genuine expressions of the political will of the lower classes without any real chances to succeed while waiting for Civil Society and the State to wither away.

**Two Types of Superstructure**

It thus turns out that there are two kinds of superstructure, a “soft,” *emergent* one that depends on the basis from which it emerges, and a “hard,” **dominant** one that converts its basis into a subsidiary with which it amalgamates. The first type is open-ended, but tenuous; the second, closed and laden with tension as dominance amalgamates with the basis that it dominates while it continues to depend on it. In sum, there exist two kinds of superstructure: the one *emergent* and dependent on the basis from which it arises (marked by an asterisk); the other, **dominant** (bold and underlined) and amalgamated with its **subsidiaries** (in bold letters). As the following table brings out:
Following our previous analyses, it should come as no surprise that the structure of the Mind is homologous with that of Society, i.e., it combines three qualitative parameters (in lower case, reading in the horizontal) with three alternative types of CONSCIOUSNESS (in upper case), each of which combines all three parameters (reading in the vertical), viz. belonging (friends versus foes), self-articulation (consent versus dissent), and moral judgment (right versus wrong).

It is by developing enduring Ingroup Feelings that are passed down through generations (mostly in fighting neighbors and enemies) that a first inchoate form of Common Consciousness develops which combines determinate Ingroup-Feelings with emergent forms of distanciation and moral judgment without dominating them. In a further step, as diffuse Definitions of the Situation gather strength and dominate Ingroup-Feelings, a new type of consciousness emerges, which distanciates the individual from his or her Ingroup
and which constitutes Status Consciousness, which includes Marxian Ideology as well as Merton’s notion of Reference Groups.

Finally, a third form of consciousness arises when people develop moral judgment of right or wrong independent of the idols of the cave, the tribe and the market: With Moral Consciousness the Mind comes into its own as Might is superseded by Right.

At the same time we note a certain affinity of each type of Consciousness with the three types of Society: Gentile Society (primitive households and clans) accords with Common Consciousness; Civil Society, with Status Consciousness, and the State, with Moral Consciousness as the basis of legitimacy.

**States of Compactness: Openness versus Closure**

As our attention switches from abstract analytics to concrete totalities (marked by upper case), we end up with another surprise: Totalities comprise varying states of social compactness which remind us of the varying firm, liquid, or gaseous states of matter. Just as ice may change to water or to steam and *vice versa*, social and mental totalities develop various states of compactness, which change depending on environmental conditions, as positivism holds, or according to mental disposition, as psychology and hermeneutics hold. In any case social action and moral judgment are shot through with randomness and elude the positivist proclivity towards strict laws.

States of social and mental compactness are therefore different from stages of evolution, which are predicated on the successive growth of *science et industrie*. To the detriment of determinism no society has ever been determined by diachronic laws. Almost from its beginning society has combined various states of compactness, nor have the more compact states wiped out the less compact ones, as illustrated by Tönnies’ duality of *Gemeinschaft* and *Gesellschaft*.

Prior to any hierarchical order the various parameters of Society, the Mind and the Intellect freely emerge from Kinship. By contrast, hierarchy presupposes dominance, which is absent in Kinship. We now understand why Sartre distinguishes two types of totality, viz. micro totalities, which are the spontaneous creation of individuals free of structural constraints, and macro totalities which are marked by structural constraints of
dependence, dominance and subsidiarity. In every case it is the emergence of structural constraints which accounts for self-regulating collective totalities.

In an ascending order of compactness the primitive Households of Gentile Society thus arise from Kinship – which mankind shares with the higher animals – without ever replacing it, just as Civil Society emerges from Gentile Society, and Political Society (the State) emerges from Civil Society without wiping out their predecessor. Society is thus the aggregate of four different states of compactness, each of which constitutes a self-regulating, as it were, *autopoietic* totality\(^4\) that is free to wax or wane, but always dependent on kinship as the last resort, i.e.,

\[
\text{Society} = \text{Praxis} \lor \text{Households} \lor \text{Civil Society} \lor \text{the State}.
\]

The same ascending order of compactness informs the states of the Mind, starting with spontaneous unrestrained preferences and Values, followed by successive types of collective Consciousness each of which is open to wax or wane, but always depending the personal Ethois as the last resort, i.e.,

\[
\text{the Mind} = \text{Personal} \lor \text{Common} \lor \text{Status} \lor \text{Moral} \lor \text{Values} \lor \text{Consciousness} \lor \text{Consciousness} \lor \text{Consciousness}.
\]

We thus end up with an additional insight which has gone widely unobserved: Whatever the constraints that may ultimately smother freedom, the primary condition of Society and of the Mind are spontaneous, unrestrained responses of individuals, from which the parameters of Society, the Mind and the Intellect emerge while they continue to depend on the former.\(^5\) Sartre was therefore right to distinguish two forms of totality, viz. individual micro totalities and collective macro totalities. The first comprise hermeneutics and phenomenology to the exclusion of structural constraints; the second focus on self-regulating collective entities which combine spontaneity with structured artifacts and constitute Hegel’s objective Mind and Husserl’s life-world as the proper subject of *verstehen* and *Geisteswissenschaft*.\(^6\)

For clarity’s sake collective totalities must not be confused with the structural parameters that determine them, but do not exhaust them. Occupational structure thus determines the Family Household, but does not exhaust it, just as property determines Civil
Society, and power determines the State without exhausting them. Whichever parameter dominates, it is always enshrined in a Cartesian product that marks totalities.

Family Households are thus founded in an increased division of labor which does not rule out increase in property and power provided the latter do not rise to dominance. A tribe may thus be rich in natural resources, but not develop a persistent property structure that recasts the occupational structure. A tribe may thus develop considerable military power, but the latter remains transitory unless it is supported by a property structure that supersedes the subsistence economy of Family Households. As Giddens (1985) has argued, the city has functioned as the power container for agrarian empires the world over. From the Tigris to the Tiber city states have thus laid the foundations for a well-organized property structure which included debt slavery, class struggles and social reforms, guilds, long-distance trade, and castes⁷ just as the rise of national states in Europe was antecedenced by the rise of cities, trade and a money economy.⁸

While the sequence of Kinship, Family Households, Civil Society and the State is marked by increasing compactness which lends itself to evolutionary interpretation, each totality continues to wax or wane in its own right. Just as water continues to change into ice or steam and vice versa, the growth and diminution of Kinship, Households, Civil Society and the State is not subject to predetermined laws, but depends on a balance that leaves a choice between increased freedom and increased effectiveness – respectively the ideal of liberals and of conservatives.

**Totalities, Historicism and Existentialism**

The preceding discussions shed additional light on Piaget’s distinction between global and analytical aspects of psychological theory and on Sartre’s distinction between micro and macro totalities, on the one hand, and strict analytics, on the other hand: While analytics makes reality transparent by reducing it to structure, totalities are self-regulating entities that exist independently of being observed but do not reveal their structure.

Therefore a dialectical relationship exists between totalities and analytics. Totalities are centered in individuals and are hence embodied in time and space, either individually, as is the case with phenomenology and hermeneutics, or collectively, as is the case with social and cultural ensembles, whose only way to tackle the welter of data is the construc-
tion of ideal types. Individual biographies and political and cultural totalities are therefore
the proper subjects for historicism as the opposite of systematicity.

Starting with the Ilias, the Upanishads and the Thora, literature has centered on bi-
ographies, which reflect the spirit of the age of which they are a part. Literatursoziologie
thus studies the Mind by combining biographic narrative with impersonal ideal types – a
methodology which is strikingly congenial with Dilthey’s and Max Weber’s predilection
for verstehen and ideal types. When Weber stated in his programmatic article on “‘Object-
civity’ in Social Science and Social Policy” – with ‘objectivity’ in quotation marks—that
he considered sociology a Wirklichkeitswissenschaft rather than a Gesetzeswissenschaft,
and when he argued against nomological laws in favor of verstehen, he was not only
pleading the case of the historicism that dominated Germany.\textsuperscript{9} He was also pleading
the case of Leibnizian monads and Sartrian totalities.\textsuperscript{10}

A glance at Husserl’s phenomenology is no less illuminating: While Husserl’s di-
chotomy of eidetic and transcendental reduction correctly reflects the division into objec-
tive structure and subjective intentionality, Husserl’s analysis was limited to logic at the
expense of formal axiology and formal praxis. With all due respect for Husserl’s emi-
nence as a logician, his idea of phenomenology had to wait for Piaget’s work on Structur-
alism (1968) and Psychogenesis (1987) and for Sartre’s (1943) existentialism.

In his later work Piaget did one more step to clarify the notion of totality by founding
it in equilibrium: Diffuse and ubiquitous, equilibrium provides the key to Sartre’s totali-
ties and to Leibniz’ monads. What accounts for the self-regulation and the unity of Pia-
get’s wholes and of Sartre’s totalities is no longer attributed to logical coherence but to
physical equilibrium. As Leibniz rightly noted, ideal constructs are devoid of force.

It now seems that we had to wait for Sartre’s (1963) attack on Marxist structuralism
to demystify Leibniz’ notion of monads and the organicism of Systems theory. For all his
flirt with Marxism Sartre attacked structuralism for failing to grasp the uniqueness of in-
dividual action and feeling and in particular, of intentionality. The latter is founded in the
effort to wrench possibility, not from Being, as classical ontology maintained, but from
Nothingness: The future is generated by devising projects that transcend not only the pre-
sent, but Being. Sartre’s existentialism thus shakes the belief in immutable substances
and replaces them with projects which are man-made and open to permanent change. To-
talities are therefore fundamentally different from formal structure. In contrast to the latter, totalities resemble the weather, which is inherently contingent and yet reducible to the changing relationship of volume, mass, and pressure.

**Two Types of *A priori*: Structural and Existential**

As Nicholas Rescher (1979, p.19) has emphasized in his work on **Cognitive Systematization**, “cognitive systematicity is the hallmark of the scientific.” At the same time Rescher is careful to distinguish cognitive from ontological systematicity. Only the first is warranted, whereas the second must be replaced by the Leibnizian principle of sufficient reason, which is inherently contingent because it is endowed with force. What Rescher rejects as “ontological systematicity” thus turns out to apply to self-regulating totalities rather than to abstract analytical systems.

The distinction between systematicity and totality leads us to discover two types of *priori* and revises Plato and Kant in unexpected ways. What Plato (*Sophist* 259e) ingeniously addressed as *symplekē tôn eidōn*, i.e., the “weaving together of forms” – i.e., of structure – and what Peirce addressed as “connections of signs with each other” thus reveals the ultimate source of rational truth and of the Kantian *a priori*, viz. systematicity, which Kant addressed (A 832) as architectonic:

> By an architectonic I understand the art of constructing systems. As systematic unity is what first raises ordinary knowledge to the rank of science, that is, makes a system out of a mere aggregate of knowledge, architectonic is the scientific in our knowledge...

The notion of systematicity adds a new, constructivist meaning to Kant’s notion of the *a priori*, which is no longer founded in ontology, but in logic. However, Kant (A 833) continues with a version that is no longer based on systematicity, but on wholeness:

> The unity of the end [that is, of the whole] to which all the parts relate and in the idea of which they all stand in relation to one another, makes it possible for us to determine from our knowledge of the other parts whether any part be missing, and to prevent any arbitrary addition… The whole is thus an organized unity and not an aggregate. It may grow from within, but not by external addition.
Kant’s idea of architectonic thus switches from formal systematization to substantive wholes, seemingly unaware that he is dealing with Leibnizian *monads*, which are linked to sufficient reason rather than formal contradiction.

Kant’s notion of architectonic thus remains ambiguous because it confounds formal systematization with substantive wholeness, whose unity is not based on logical consistency, but on contiguity, continuity and equilibrium. Hegel’s famed dictum “die Wahrheit ist das Ganze,” which inspires his entire philosophy, is thus predicated on totalities. In terms of Kant, Hegel is not an analytic but an historic thinker. Accordingly, Kant’s (A836) dichotomy of rational knowledge, which is *cognitio ex principiis*, and historical knowledge, which is *cognitio ex datis*, also applies to the notion of the *a priori*.

It thus dawns on us that there exist two forms of knowledge *a priori*, the first, structural and objective, but abstract; the second, subjective, perspectival and concrete. Since Kant had no idea that there exists a second type of *a priori*, the *transcendental Ego* remained unaccounted for in the *Critique*. Fichte, its first consistent advocate, could rightly claim that he had understood transcendental philosophy better than Kant himself. As Fichte put it, “*Das Ich setzt das Nicht-Ich*,” i.e., passive perception is replaced with active intentionality, and the Ego thus resembles Leibniz’ windowless *monads*. On the whole, philosophy had to wait for Sartre to discover “totalization” as the second, subjective form of the *a priori*, which is wedded to the concrete here and now and to the unrelenting dialectic of the Ego and the projects that transcend it.

### Three Stages of Social Theory

With Sartre’s notion of concrete micro and macro totalities, on the one hand, and abstract analytics, on the other hand, we arrive at a tripartite division, which is crucial not only for sociology, but for the social and cultural sciences in general. It is not too much to say that it revises almost all extant dichotomies as either incomplete or incorrect, while it vindicates Leibniz and Hegel and sheds new light on Dilthey’s concept of *Geisteswissenschaften* and on historicism.

For simplicity’s sake, let us start with analytics and its reduction to structure, which produces objective, “rational” truth *a priori*, not on the basis of mathematization and formalization, but on the basis of set theory, which reduces reality to distinctive qualities
as mapping functions. It then becomes clear that *eidetic* reduction is tantamount to structural reduction and by the same token, to a farewell to metaphysics: Rather than inherent on the object, structure is a creation of the human intellect.

Strictly speaking, it is thus construction, or even more precisely, constructivism rather than structure that defines analytics and equates it with strict science and mathematics. Analytics must therefore not be confused with structuralism as a special type of sociological theory that focuses on social stratification to the exclusion of individual action and personal values, which in terms of Sartre fall into the realm of totalities.

At the same time, structuralism claims analytic status because it reduces manifold action to unified structure. It is for this reason that structuralism represents a powerful type of social theory. Starting with Montesquieu and Rousseau it lies at the bottom of St.-Simon’s, Comte’s and Durkheim’s positivism as well as of Marxism, with Althusser, Peter Blau and James Coleman as its leading proponents.

Whatever the merits of structuralism, it is obvious that it is secondary to spontaneous action in the sense of C.H. Cooley’s primary groups and ideas and of G.H. Mead’s Self. A distinction is thus made between pristine action, which is uninhibited by rules, and action “following a rule,” a distinction which has been the subject of an intense discussion of *The Micro-Macro Link* (Alexander et al. 1987). For intrinsic reasons the latter did not result in a general consensus. On the one hand, the micro-macro dichotomy must not be confused with the division between objective status and subjective values. In addition the dichotomy reverberates in Sartre’s distinction between (personal) micro totalities and (collective) macro totalities, a dichotomy which is much more to the point but which does not seem to have reached beyond Sartrian existentialism although it has two prominent predecessors in Hegel’s duality of subjective and objective mind and in German historicism.

The duality of micro and macro totalities also has two unacknowledged parallels, viz. Durkheim’s (1957) distinction between *Professional Ethics and Civic Morals* and Parsons’ duality of Personality Systems and Culture Systems. Unrecognized by Parsons, the two systems share the same parameters, viz. Ingroup Feeling, Definition of the Situation, and Sense of Justice, with the important difference that Personality Systems deal with micro totalities such as the Self, the family and the *gens* while Culture Systems deal with
macro totalities such as nations and civilizations, historical epochs, art styles etc. What Parsons took for two separate domains addresses two types of totality, the one intrapersonal, the other, suprapersonal, as manifest in *Zeitgeist* and *Völksgeist*. Both types of totality constitute wholes, albeit in different ways, the one closed and centered, the other, open and without a special agency to control its unity.

Micro totalities are thus akin to *monads*, which are centered in the Ego. Accordingly they comprise a world that is centered in the here and now even when the Ego memorizes the past or devises projects that anticipate the future. In terms of Mead, the Self is the Cartesian product of the (present, relentless) “I” and of its past projects, the “me.” While it is predicated on intentionality that transcends the present, the Self is a closed totality whose hallmarks are self-regulation and its uniqueness.

Note, however, that self-centeredness is not identical with isolation. While it is true that the Self reduces the interaction with its partners to the projects it designs with regard to them and thus creates the world in which he or she lives, it is still true that this “life-world” does not imply isolation any more than do Leibniz’s *monads*. The latter are windowless in the sense that for them the world exists only so far as they react to it. What counts for the Selfs that interact is not interaction *per se*, but the diverse projects that guide their action. As far as micro totality is concerned, totalization is identical with reduction of the Self to intentionality.

By contrast, macro totalities combine the projects of all the Selfs that interact, as Rousseau’s (1762) path-breaking essay on *The Social Contract* illustrates. Being predicated on contract theory, Rousseau’s could have avoided the realistic fallacy of believing in suprapersonal entities had he not credited the general will with a compact unity it does not possess. By contrast, Sartre offers a nominalist interpretation of macro totalities that replaces static totality with totalization as a process that is never completed because of the indelible freedom of the interacting Selfs. According to Sartre’s own premises the latter act as detotalizing factors that obviate the closure of macro totalities. In Sartre’s own terms macro totalities are totalities without a totalizer.
The Intricacies of Macro Totalities

We thus arrive at an interpretation of macro totalities that seems to converge with Durkheim’s concept of collective representation according to which a society is the more integrated the more its members share common values. Unfortunately this interpretation fails to explain the thrust, or in terms of Leibniz, the force, that distinguishes monads from abstract sets and systems that are based on eidetic reduction. The question thus arises what accounts for the momentum that distinguishes macro totalities from mere collectivities, on the one hand, and from micro totalities, on the other hand.

The answer goes back to Hegel: Ideas move history if they articulate the spirit of the epoch. The latter is the case with projects that are the creation of individual Selves which are embedded in space and time but at the same time transcend the given situation and provide leadership that tackles the problems the given society is facing – a process that is illustrated by the Declaration of Independence of the first Continental Congress and by every presidential election in the United States, when public opinion is shaken up by competing projects which vie for public support.

Macro totalities thus differ fundamentally from micro totalities. On the one hand, they are composed of micro totalities rather than of individual agents; on the other hand, they lack the unity of micro totalities because they lack a supra-personal agent in the vein of an imaginary collective Self as the source of suprapersonal projects. In every case Zeitgeist and Volksgeist are the outcome of interacting micro totalities, which are in turn composed of individual Selves. In contrast to micro totalities, which are limited to interacting Selves, macro totalities are predicated on impersonal projects calculated to muster public support and if accepted, to function as constraints imposed on the collective.

We now understand why Sartre speaks of “totalization without a totalizer.” While he correctly, but perfunctorily points to institutions, his perception is blurred by his broad use of the “practico-inert,” which comprises input from nature as well as output from society and thus blurs the difference between natural resources, which are subject to entropy, on the one hand, and the production of artifacts, whose function it is to reduce entropy, on the other hand. The point is that micro totalities depend on their natural environment, which they selectively convert into a subsidiary, while macro totalities are the outcome of public projects which are predicated on the invention and use of artifacts. It is
the artifacts – not only laws and institutions, but also tools and weapons, buildings, roads and canals, carriages and boats etc.—which function as the totalizers that distinguish macro totalities from micro totalities.

A glance at the American electoral system illustrates this. Given the (external, institutional) constraint to choose between two candidates, the particularistic interests of the Selves are relegated to subsidiarity while the projects of the contending candidates function as totalizers. It is in this sense that the contending parties represent macro totalities that appeal to society as a whole while they are “detotalized” by the individuals that disagree. Contrary to what the term “totality” suggests, nothing is definitive so long as individuals are free to devise new projects.

What distinguishes macro totality from micro totality is not the existence of two different types of Selves, one micro and one macro, but the constraints of institutions which account for the functionality and rationality of macro totalities without sacrificing the nominalism that distinguishes Sartre’s existentialism from the realism that underlies Leibniz’ monads as well as Hegel’s objective mind. As individual wills with different pasts and different projects compete and cooperate under given economic, social and political conditions, the projects that make the most of these conditions – i.e., that promise optimal effects with minimal effort—will win maximal support and thus constitute macro totalities without involving inexplicable final causes.

While there is no doubt that all macro totality is driven by individual Selves, macro totalities differ from micro totalities by the invention and use of artifacts such as technology, investment, language and law, which subject the infinitely active Selves to functionality and rationality as the hidden totalizers that integrate polythetic Selves into monothetic Societies, Nations, Religions and Civilizations. What structure is unable to achieve, viz. finality and dynamics, macro totalities do achieve. By the same token they reveal the ultimate secret of historicism and its inherent realism: Macro totalities are inherently unique because they are the work of Selves which are inherently here and now.
Real Monads and Ideal Systems

In the light of the preceding analyses Leibniz’ idea of monads gains new weight and importance, especially if we interpret monads as macro totalities which combine subjective intentionality with objective rationality. In particular the notion of factual truth calls for a revision that no longer equates factual truth with isolated facts but with wholeness (totality), concreteness (here and now) and singular uniqueness, as embraced by historicism, *Wirklichkeitswissenschaft* (Max Weber, Hans Freyer), American pragmatism, and French existentialism (Bergson, Piaget, Sartre) and Heidegger’s fundamental ontology, all of which culminate in the discovery of the dual notion of the *a priori*. As Leibniz put it (*Monadology* 1998, p. 272),

Our reasonings are founded on two great principles: *the principle of contradiction*… and that of *sufficient reason*… When a truth is necessary, the reason for it can be found by analysis, by resolving it into simpler ideas and truths until we arrive at the basic ones…in a word, *basic principles* which can never be proved, but which also have no need of proof: these are *identical propositions*, the opposite of which contains an explicit contradiction…

But a *sufficient reason* must also be found for *contingent truths*, or *truths of facts*, for the series of things which fills the universe… Here the resolution into particular reasons could be continued endlessly, because of the immense variety of things in nature… There are an infinite number of shapes and of motions, present and past, which play a part in the efficient cause of my present writing; and there are an infinite number of tiny inclinations and dispositions of my soul, present and past, which play a part in its final cause.

The preceding quote establishes Leibniz as one of the pivotal thinkers of modernity who realized that systematicity is limited to ideal constructs which are ultimately founded in logic, but like shadows are devoid of power. By contrast, factual truth is dynamic because it is centered in the transcendental Ego, which is always here and now. We thus end up with two kinds of *a priori*, the first based on objective, ideal noemata; the second based on actual, real noeseis; the first based on *eidetic* reduction and aimed at objective structure; the second based on *transcendental*, intentional reduction and inherently
unique and perspectival; the first, constitutive of rigorous science; the second, constitutive of historicism and existentialism.

Leibniz’ dichotomy has had three prominent successors, viz. Vico’s (1744) dichotomy of the *verum* and the *certum*, for which Vico has been credited with founding historicism; Schopenhauer’s (1818) opposition of Will and Idea, which anticipates Nietzsche and Freud; and Husserl’s (1913, 1929) dichotomy of eidetic and transcendental reduction, which has brought Leibniz’ and, incidentally, Kant’s thought to full fruition.

Leibniz’ dichotomy also reverberates on sociological theory. While theory in the strict sense is defined by systematicity – i.e., consistency, completeness, and simplicity—action is wedded to the here and now, implying contingency, uniqueness and irreducibility. By the same token totalities are bound to produce their own type of order. For all its uniqueness action is bound to attain equilibrium as a precondition for stability. In the last analysis equilibrium is the hidden force that makes sure the world in which people live is in Leibniz’ terms “the best of all possible worlds.” What Voltaire ridiculed as a philosophical crotchet thus reveals the principle that rules totalities.

Accordingly, the business of sociology falls into two sectors which work on opposite principles: social theory and social research. The first is based on logic – Leibniz’ principle of contradiction—which culminates in systematicity as the Cartesian product of coherence, completeness and simplicity (parsimony). The second is based on action, which is predicated on the here and now even where it anticipates the future. However intricate its methodology, social research is limited to the sampling of data, which are inherently contingent because they deal with concrete totalities here and now rather than with ideal systems which are predicated on logic. Again, Leibniz’ dichotomy works.

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Endnotes

1 It is in this sense that all meaning is “transcendental” and intentional.

2 It thus elucidates that qualitative sociology is identical with analytical sociology.

3 As Tönnies had subsequently to acknowledge, Gesellschaft never fully replaces Gemeinschaft, as he had assumed in the first edition (1887).

4 It thus elucidates that Luhmann’s theory, like Systems Theory in general, is predicated on totalities and at the expense of detail which only structural reduction can provide.

5 Symbolic interactionism is therefore vindicated if it argues against structuralism that action is not reducible to structure.

6 Let us note that a close personal relationship existed between Dilthey and Husserl, who considered Dilthey’s work a kind of phenomenology.

7 For an exhaustive study see Max Weber’s (1976) masterpiece on The Agrarian Sociology of Ancient Civilizations.

8 For a book-length study of the City see Max Weber’s (1978) chapter xvi in Economy and Society.

9 As Weber (1949, p.72) put it, “The type of social science in which we are interested is an empirical science of concrete reality (Wirklichkeitswissenschaft). Our aim is the understanding of the characteristic uniqueness of the reality in which we move.”

10 While there is no doubt that Weber was influenced by Rickert’s notion of Kulturwissenschaft, the reference to the notion of totality and to the uniqueness of the reality in which we move make it clear that verstehen is not limited to value judgment, but extends to intentionality in general, including both purposive and normative rationality.

11 Strictly speaking there are only two macro totalities which cannot be “detotalized:” the personal Fatum and Equilibrium, i.e., Deus sive natura.