THE LOGICAL FOUNDATIONS
OF ANALYTICAL SOCIOLOGY

by

Gert Harald Mueller
The American University
Washington, D.C.

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THE ANALYTICAL FOUNDATIONS OF LOGIC

The Dual Composition of Logic

The preceding chapter has shown the intricate link of analytical theory with Ontology. Once the latter is freed of its Platonic legacy it is rehabilitated as the precursor of modern structuralism. As the comparison of implication \(a \rightarrow b\) and presupposition \(a \leftarrow b\) has shown, the power of logic is all pervasive. Theory has no longer to wait for mathematics to become a strict, “exact” science, and what recent discussion has touted as “linguistic turn” turns out to be the turn to logic as it is bound up with language as its carrier.

At the same time the notion “applied logic” raises doubts because it reflects a deeply ingrained Platonism in our thought. While there exists indeed such a thing as “pure” (formal) logic, the latter is a derivative of the former, as the pragmatist revolution of “radical empiricism” has made clear. In order to emerge formal logic presupposes ordinary language, i.e., Speech. Rather than founding language in logic, the reverse is the case, i.e., Speech represents the primal branch of semiology from which formal logic is distilled. In fact, every identification implies the denial of its negation according to the formula

\[
(a = a) = \neg(a = \neg a). \quad \rightarrow \quad \neg(a = b). = (a = b) = FFFF.
\]

The same holds for the constitution of language in early childhood, a tack pursued by Piaget and Lévi-Strauss, who noted the contrast of opposites in constituting the child’s Speech: Just as he has to distinguish turning right from turning left, the child starts discerning reality by distinguishing warm from cold, big from small, push from shove, friend from foe etc., even before acquiring language. In fact all these elementary distinctions are shared by humans with the higher animals.

In every case human speakers and animals are equally unaware of the four truth-values that define every truth-function, and yet they think accurately. In the upshot Erkenntnistheorie reduces to what Kant might have called “transcendental Logic” – the term actually used by Husserl (1929) in contrast to formal Logic. The latter produces pure Logic, but blocks cognition because it works with anonymous variables. It is by using “saturated,” substantive variables that “transcendental Logic” produces truth.
Truth-Conditions and Truth-Functions

A clear distinction must therefore be made between arguments and functions. The first are substantive and refer to reality; the second are syntactic and are defined by their truth-values. Arguments and functions may therefore alter independently, as the various combinations of bread and butter, tea and coffee etc. easily show. What is less realized is that there exists an important difference between truth-functions, on the one hand, which connect substantive, “saturated” variables, and purely syntactical truth-conditions, on the other hand, which work with “unsaturated,” algebraic variables and block cognition – the dual reason why positivist logistics and Kantian Erkenntnistheorie are equally blind to the link that underlies all logic as well as all cognition.

It is therefore all the more remarkable that Wittgenstein (TLP: 4.01) chimed in with Kantianism. As he put it, "The proposition is a model of reality such as we conceive it." Whatever the relations between "things in themselves" and the abstruse juggling with noumena, it is the subsumption of substantive arguments under a truth function which constitutes logical, or rational truth a priori. As Wittgenstein (TLP 4.463) put it, "The truth-conditions of a proposition determine the range that it leaves open to the facts." It is the subsumption under truth-conditions which “makes strict science possible” while avoiding Kant’s convoluted notion of the a priori.

Logical Quadrants

The preceding analyses give a new answer to the Kantian question, How is science possible? Science transcends empirical truth if its propositions satisfy the Wahrheitsbedingungen of a truth function, i.e., as Wittgenstein put it (TLP 4.463), "Die Wahrheitsbedingungen bestimmen den Spielraum, der den Tatsachen durch den Satz gelassen wird." (The truth-conditions of a proposition determine the range that it leaves open to the facts.)

The question then arises how pure, seemingly mathematical logic is constituted. The answer is that there are exactly sixteen truth functions, which are ruled by a strict regime of consistency, simplicity (parsimony) and completeness which makes all truth functions convertible to one another so long as their truth-values are maintained.
THE FIRST QUADRANT: SYMMETRY AND COMMUTATIVITY

Zero-Form, Joint Denial, Converse, and Dual

Let us turn to the first of the four quadrants, which is the one that is most broadly known and easiest to interpret. It is characterized by two distinctive qualities: symmetry and commutativity. Most important, a strict order rules among the four functions, each of which relates to the others either as denial, converse, or dual.

(1) For obvious reasons, global, joint denial is the most elementary function of logic that divides truth functions into contradictory pairs which mutually define each other:

\[
\begin{align*}
A \lor B &= \neg(A \downarrow B), \\
A \downarrow B &= \neg(A \lor B), \\
A \land B &= \neg(A \mid B), \\
A \mid B &= \neg(A \& B).
\end{align*}
\]

(2) Along with joint denial, there exists a converse to every truth function, which is marked by the negation of its arguments, i.e., by dual partial denial:

\[
\begin{align*}
A \lor B &= \neg A \mid \neg B, \text{ known as } modus ponens. \\
A \mid B &= \neg A \lor \neg B, \text{ known as } modus tollens. \\
A \& B &= \neg A \downarrow \neg B, \text{ known as } modus ponens. \\
A \downarrow B &= \neg A \& \neg B, \text{ known as } modus tollens.
\end{align*}
\]

(3) Thirdly, we may combine global, joint denial and dual partial denial, which results in the dual, as known as De Morgan's law, viz.

\[
\begin{align*}
A \lor B &= \neg(\neg A \& \neg B). \\
A \& B &= \neg(\neg A \lor \neg B).
\end{align*}
\]

The dual is not limited to alternation and conjunction. It also holds for joint denial (\(\downarrow\)) and incompatibility (\(\mid\)):

\[
\begin{align*}
A \downarrow B &= \neg(\neg A \mid \neg B), \\
A \mid B &= \neg(\neg A \downarrow \neg B).
\end{align*}
\]

We thus arrive at a stringent schema of mutually convertible truth functions:
Table 1
Zero-Form, Joint Denial, Converse, and Dual

<table>
<thead>
<tr>
<th>Zero Form</th>
<th>Joint Denial</th>
<th>Converse</th>
<th>Dual</th>
</tr>
</thead>
</table>

The consistency, perspicuity and stringency of the regime which rules the four functions is self-explanatory. Whatever the complexity of its form, each function is reducible to its normal form which presents the meaning of the function in the most straightforward and simplest form.

The first column shows each truth function in its pure, or zero-form, i.e., unencumbered by global or partial denial. For its purity and transparency we may also call it the normal form. It is also the form that is closest to everyday language.

The second column connotes the global, or joint denial of the normal form. It tells us what the function rules out. As it were, it sorts out the bad apples without saying much about the good apples.

The third column represents the converse of the zero-form. Unlike joint denial, it is specific about the arguments which it negates and thus spells out the logical implications of the normal form. For example, while the global denial

\[ A \lor B. = \sim (A \downarrow B), \]

looks somewhat contrived, the converse is forthright and explicit; it easily carries over to the dual, e.g.,

\[ A \lor B. = \sim A \mid \sim B. = \sim (~A \& ~B). \]

Finally, the fourth column conveys the dual to the zero-form. It is known as De Morgan's law and is broadly used for the mutual conversion of the conjunctive and the disjunctive function. Otherwise, it adds little to what is already spelled out by the converse,
e.g.,

$$A \& B. = \sim A \downarrow \sim B,$$

which reads “neither non-A nor non-B.” In every case, zero-form, denial, converse and dual define each other and constitute a self-explanatory, closed logical quadrant.

The preceding analyses have important implications:

(a) Whatever its complexity, any logical matrix is reducible to zero-form, which reveals its syntactical meaning.

(b) Any change beyond the conversion rules changes the meaning of the truth function and thus implies a switch to a different truth function with a different syntax.

(c) In spite of its rigor, logic invites Ideenvariation (Husserl 1913), inventiveness and creativeness. Like the arts, it is intellectually liberating and sensitive to elegance.

(d) Each function exists in its own right. While functions are convertible, they are not reducible to each other.

### The Pitfalls of Formal Logic

The previous analysis of pure functions regardless of substantive content reveals the power and flexibility that inheres in the functions as a necessary, but not sufficient condition of human thought (Frege 1921): Pure logic produces schemata, but not truth. To produce the latter, and that is, to convert schemata into propositions, substantive arguments must complement the functions.

The cognitive role of logic is obscured and the syntactical meaning of the functions is outright lost if logic is reduced to a mere technique of shuffling truth-values over a universe which is reduced to "p" and "q." Whether a truth function is applicable only a glance at the intended meaning and the arguments can tell. To be fully understandable, syntactic meaning must combine with substantive meaning.\(^1\)

The emphasis on substantive truth implies the switch from formal, algebraic logic to substantive, applied logic in tandem with the switch from propositions to terms. As the discussion of sodium and chlorine has shown, terms rather than propositions are the hub of logic. In fact switching from propositional logic to terminal logic will spare as statements such as, e.g., "If Paris is the capital of China, squares are round" (Quine 1982, p.24).\(^2\)
Far from being exemplary, statements of this sort represent a misuse of logic. They easily collapse if the propositions are substituted with terms: Even if it were true, e.g., that "Paris is the capital of France," and that "Squares are rectangular," it would still not be true -- and that is what the schema "If p, q" states -- that Paris implies squares. Nor does the argument fare any better if the underlying matrix is reinterpreted as "¬p→¬q", whose zero-form, "p ← q," is equally inapplicable. Squares do not presuppose Paris as a necessary condition any more than they are a sufficient reason for it. Obviously, the two have nothing in common. It thus stands to reason that neither implication nor presupposition apply.

The question still remains how a true statement about Paris and about squares should look like. The answer is: "It is not true that Paris is the capital of China or that squares are round." We have no difficulty to identify the zero-form which underlies this statement:

\[ ¬(p \lor q) = ¬(¬p \mid ¬q) = ¬p \& ¬q = p \downarrow q, \]

which reads: "Neither is Paris the capital of China, nor are squares round." The point is that the use of "if" in the chosen example results in a nonsensical statement because it runs counter to the implied meaning of the statement, which is, "if someone is ignorant enough to believe that Paris is the capital of China, he might as well believe that squares are round."

**Formal and Substantive Logic**

Our criticism of misapplied logic does not stop here. Our case can be argued on strictly formal grounds as well. Granting that arguments p and q are both false, nothing warrants strict implication (→) as the pertinent truth function. Of the sixteen truth functions no less than eight allow for the combination of ¬p and ¬q, viz.

<table>
<thead>
<tr>
<th>p ↓ q</th>
<th>p ↑ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>p → q</td>
<td>p ← q</td>
</tr>
<tr>
<td>p ≠ q</td>
<td>p ↔ q</td>
</tr>
<tr>
<td>N ¬p, P q v ¬q</td>
<td>P p v ¬p, N ¬q</td>
</tr>
</tbody>
</table>

The point is that nothing warrants the use of isolated truth-values. Without exception it takes all four truth condition, i.e., ++, +−, −+, −−, to define any of the sixteen truth-functions, which in the present case is indubitably FFFT.
**Everyday Speech and Substantive Logic**

The switch from formal logic to substantive logic and to the inclusion of all sixteen truth-functions enhances the scope and power of logic and reconciles it with ordinary, everyday Speech.

<table>
<thead>
<tr>
<th>A v B</th>
<th>A ⊁ B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inventory</strong></td>
<td><strong>Summary Denial</strong></td>
</tr>
<tr>
<td>Enumeration</td>
<td>Rejection</td>
</tr>
<tr>
<td><strong>Incompatibility</strong></td>
<td><strong>Complementarity</strong></td>
</tr>
<tr>
<td>Exclusion</td>
<td>Inseparability</td>
</tr>
</tbody>
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<tr>
<td>Exclusion</td>
<td>Rejection</td>
</tr>
</tbody>
</table>

In every case ordinary Language, i.e. Speech, combines Logic with substantive content:

- Adjunction maps inventory, enumeration, contiguity, alternation, free choice etc.;
- Joint denial maps summary denial, rejection, rebuttal, dismissal etc.;
- Disjunction maps incompatibility, exclusion, decision, strict choice etc.;
- Conjunction maps complementarity, inseparability etc..

In other words, while formal logic deals with algebraic arguments such as A, B, etc., each function is marked by distinctive syntactic meaning regardless of the semantic meaning of the arguments which it connects. Its formal character notwithstanding Logic is therefore distinctly different from mathematics. It is by no means a subset of mathematics, let alone of physics, as logistics and the Viennese Circle would contend.

On the other hand, while the four functions are convertible to each other, any conversion is legitimate only if the truth-values which define each function remain unaltered. To reduce logic to a few functions such as adjunction (v), conjunction (&), and implication (→) is therefore to curtail Logic to the point of impotence.
THE SECOND QUADRANT: ASYMMETRY AND IRREVERSIBILITY

The first quadrant of truth functions is followed by a second quadrant which is no less impressive for its consistency and self-sufficiency. Rather than on symmetry and commutativity it is predicated on asymmetry and irreversibility. By the same token its functions combine necessity of one of its arguments with open-endedness of the other argument.

**Implication and Replication**

As a standard procedure in propositional logic, implication and its converse, replication (i.e., presupposition) are subsumed under the common denominator of "conditionals," thus blurring the fact that the two functions are quite different. To be sure, nobody disputes that

\[ A \rightarrow B. = A \mid \neg B. = \neg A \lor B. = \neg (A \land \neg B) \]

is not identical with

\[ A \leftarrow B. = \neg A \mid B. = A \lor \neg B. = \neg (\neg A \land B). \]

Confusion sets in if the sequence of the arguments is reversed on grounds of proof theory. Arguing that

\[ A \rightarrow B. = \neg A \lor B. = A \mid \neg B \]

is equivalent to

\[ B \leftarrow A. = B \lor \neg A. = \neg B \mid A. \]

implies a gross fallacy, which becomes apparent when we translate the two schemata into ordinary language. "A→B" connotes "A implies B", or more forcefully, "B is a necessary consequence of A". By contrast, its inversion, "B←A" reads "A presupposes B," or "B is a necessary condition for A," which leaves us with a perplexing contradiction: How can the necessary consequence of A be at once its necessary condition, and conversely, how can the antecedent of a necessary consequence be possible only if the consequent is already presupposed? The point is that in contrast to formal, allegedly “mathematical” Logic substantive, material Logic does not allow for inversion, i.e.,

\[ \neg A \lor B. \neq B \lor \neg A. \]
**Intensio Directa and Intensio Indirecta**

The problem does not lie with the imprecision of ordinary language, which remains remarkably truthful to reality. Nor is the problem to be attributed to logical indeterminacy. Rather, it lies with the improper use of truth-values. To illustrate our point, let us substitute "cause" (CAU) for A, and "effect" (EFF) for B. Then it is obvious that

\[
\text{CAU} \implies \text{EFF}. = \sim (\text{CAU} \& \sim \text{EFF}). = \text{CAU} \mid \sim \text{EFF},
\]

i.e., "Cause entails effect as a necessary consequence" is equivalent to "It is not true that there is a cause and no effect," and to "The assumption of a cause excludes the denial of an effect." Wherever there is a cause, there must be an effect.

At the same time, the inversion "EFF $\leftarrow$ CAU" is not bare of sense: Awareness of a cause does indeed presuppose awareness of an effect. While both statements deal with the same *Sachverhalt*, CAU => EFF addresses reality *intensio directa* whereas "EFF $\leftarrow$ CAU" addresses it *intensio indirecta*. The two matrices are therefore quite different in character: The first is a statement, the second, an inference which presupposes some material statement. Obviously inversion of the arguments is not permissible for the second quadrant because it confuses statements *intensio directa* with inferences *intensio indirecta*.

In order to avoid confusion, let us mark off the direct mode with a double-shafted arrow (⇒), and the indirect mode, with a single-lined arrow (→) and quotation marks to denote *intensio indirecta*. In this manner A ⇒ B denotes necessary consequence, while "B $\leftarrow$ A" denotes presupposition inferred from it, i.e.,

\[
\text{A} \Rightarrow \text{B}. \rightarrow "\text{B} \leftarrow \text{A}".
\]

Likewise, A $\leftarrow$ B denotes foundation, or necessary condition, while "B $\rightarrow$ A" denotes strict implication inferred (→) from the foundational formula, i.e.,

\[
\text{A} \leftarrow \text{B}. \rightarrow "\text{B} \rightarrow \text{A}".
\]

In sum, double-shafted arrows denote objective statements. By contrast, single-shafted arrows denote inference derived from the former. A sharp distinction must therefore be made between the direct and the indirect mode, and by the same token, between "substantive" logic, which works with saturated variables and produces truth, and formal, algebraic logic, which works with unsaturated variables and does not produce truth.³
Entailment and Necessary Consequence

To illustrate our case, let us substitute causality for strict implication. Then it is evident that once there is a cause (CAU), there must be an effect (EFF), i.e.,

\[ CAU \Rightarrow EFF = \neg( CAU \& \neg EFF) = CAU \mid \neg EFF. \]

At the same time, while inversion of the arguments is not permissible, conversion without changing the sequence of the arguments is legitimate: The equation

\[ CAU \Rightarrow EFF = \neg CAU \iff \neg EFF \]

stays in the direct mode (with the sequence of terms unchanged) and makes good sense: If a determinate cause entails a determinate effect, eliminating the cause is a necessary condition for avoiding the effect. (Note that the absence of a cause is not a fact, but an a conjecture):

\[ \neg CAU \iff \neg EFF \neq \neg EFF \Rightarrow \neg CAU. \]

Obviously it would be odd to assume that an inexistent effect could entail an equally inexistent cause. In sum, truth-functional appearance to the contrary, the equation

\[ CAU \Rightarrow EFF = EFF \iff CAU = \neg EFF \Rightarrow \neg CAU \]

is false both on material and formal grounds: Materially, it is nonsensical; formally, it violates the rule that forbids inversion of the functions of the second quadrant.

However, the inversion becomes meaningful if we switch from the direct to the indirect mode, i.e.,

\[ CAU \Rightarrow EFF \rightarrow \neg EFF \rightarrow \neg CAU. = \neg EFF \leftarrow CAU. \]

It certainly makes sense to infer that awareness of an effect is the necessary condition for searching for a cause. Indeed, looking for a cause presupposes not so much an effect than the awareness of an effect.

The distinction between direct and indirect mode has far-reaching consequences for the perspicuous understanding of Logic, for direct, “material” implication is not identical with “strict” implication. The latter is strict, not on ontological grounds (viz. empirical evidence), but on grounds of being a logical derivate of other functions which are founded in direct evidence such as necessary consequence, necessary condition and, as we shall see shortly, randomness and independence, as well as conjunction, disjunction, etc.
Foundation and Necessary Condition

As already noted, the conditional truth function follows the same principles as entailment, its converse. To illustrate our case, let us assume that the law forbids people under 18 years of age to drink beer in public places. Then it is obvious that being under the age of 18 rules out drinking beer and that being 18 is a necessary condition for getting beer, i.e.,

\[
\neg \text{Age18} \Rightarrow \neg \text{BEER.} = \neg \text{Age18} | \text{BEER.} = \text{Age18} \Leftarrow \text{BEER.}
\]

It is obvious from this equation that the counterfactual is the most effective way to test whether a conditional is true. By the same token, the distinction between the direct and indirect mode is as crucial for conditionals as it is for entailment. Inverting the sequence of the arguments would be tantamount to saying that drinking beer entails becoming 18 years old, or that abstaining from beer is a necessary condition for being under 18, i.e.,

\[
\text{Age18} \Leftarrow \text{BEER.} \neq \text{BEER} \Rightarrow \text{Age18.} = \neg \text{BEER} \Leftarrow \neg \text{Age18},
\]

all of which border on the nonsensical. However, seeing people drink beer in public we may infer that they are 18 years of age, i.e., getting beer strictly implies, but does not materially imply, i.e., entail, being 18, i.e.,

\[
\text{Age18} \Leftarrow \text{BEER.} \rightarrow \text{“BEER} \rightarrow \text{Age18.”} \neq \text{BEER} \Rightarrow \text{Age18.}
\]

In every case, the distinction between the direct and indirect mode, i.e., between \textit{intentio directa} and \textit{intentio indirecta} accounts for the difference between material foundation (\Leftarrow) and strict implication (\rightarrow). As in the case of causality the distinction between the direct and the indirect mode is fundamental for understanding Logic – a requirement which most logic books do not meet, which reduce the difference between “bound” and “unbound” variables to a mere technicality.

At the same time the role of strict implication is easily demonstrated by the elementary fact that each of the sixteen truth-functions is convertible not only to its joint denial, converse and dual, but as Nicod and Sheffer have shown, to any other of the sixteen truth-functions \textit{salva veritate}, however complex the formula. For example,

\[
\begin{align*}
\text{A v B.} &= (\neg\text{A} \rightarrow \text{B.} \& \neg\text{B} \rightarrow \text{A}) \& (\neg(\neg\text{A} \rightarrow \text{B.}) \& (\neg\text{B} \rightarrow \text{A}). \\
\text{A & B.} &= (\text{A} \rightarrow \text{B.} \& \text{B} \rightarrow \text{A}) \& (\neg(\neg\text{A} \rightarrow \text{B.}) \& (\neg\text{B} \rightarrow \text{A}). \\
\text{A \leftrightarrow B.} &= (\text{A} \rightarrow \text{B.} \& \text{B} \rightarrow \text{A}) \& (\neg(\neg\text{A} \rightarrow \text{B.}) \& (\neg\text{B} \rightarrow \text{A}). \\
\text{A opp B.} &= (\text{A} \rightarrow \neg\text{B.} \& \neg\text{B} \rightarrow \text{A}) \& (\neg(\neg\text{A} \rightarrow \text{B.}) \& (\neg\text{B} \rightarrow \text{A}).
\end{align*}
\]
Randomness and Independence

For good reasons implication stands at the center of propositional logic: It seems to be the mechanism which imparts necessity to our thought, whereas the denial of necessity seems foreign to strict thought and has met with scant attention. All that seems to be known about it in the work of Frege (1922) or Wittgenstein (1921) are the truth-values FTFF and FFTF and their identification as "A&~B" and "~A&B," which say nothing about their zero-form, i.e.,
\[ A \& \neg B. = \neg (A \lor B). = \neg (A \Rightarrow B), \]
respectively,
\[ \neg A \& B. = \neg (A \lor \neg B). = \neg (A \Leftarrow B). \]

What the conjunctural and the disjunctural formulas do not bring out, the joint denial of necessary condition (foundation) and of necessary consequence (entailment) bring to the fore: To say that A does not entail B is to say that A has no impact on B and that B is accidental to A (A acc B). In sum, the denial of entailment implies randomness, coincidence, or chance, which concurs with Frege's and Wittgenstein's definition:
\[ A \text{ acc } B. = \neg (A \Rightarrow B). = \neg (A \mid \neg B). = A \& \neg B. = \text{ FTFF} \]

In a similar fashion the denial of necessary condition defines independence: To deny that A is a necessary condition for B is to say that B is independent of A.

By the same token independence is the converse of randomness:
\[ A \text{ ind } B. = \neg (A \Leftarrow B). = \neg (\neg A \mid B). = \neg A \& B. = \text{ FFTF} \]

While the two schemata, "A&~B" and "~A&B" are silent about reality, randomness and independence represent substantive, ontological categories. Together with entailment and foundation they constitute the second quadrant as a self-explanatory domain of logic.

Most important, Logic is no longer hooked up with determinism. The second quadrant reveals the truth-functional foundations of necessity. Longstanding beliefs to the contrary, necessity is not a metaphysical category, but a logical category, whose denial explains randomness and independence. As modern physics has fully brought out, indeterminacy and rupture, spontaneity and discontinuity rule the physical cosmos as well as human thought. In every case the functions of logic underlie our ontological categories as the primal instruments of cognition.
The Second Quadrant as a Whole

Except for the basic difference between symmetry and asymmetry, the second quadrant follows the same principles of mutual convertibility as the first quadrant.

### Table 3

<table>
<thead>
<tr>
<th>Zero-Form:</th>
<th>Denial:</th>
<th>Converse:</th>
<th>Dual:</th>
</tr>
</thead>
</table>

At the same time the asymmetric functions are convertible into symmetric functions, which are much easier to handle. Thus entailment and causality easily translate into, and are defined by ~A v B = A | ~B. = ~ (A & ~B), just as hierarchy and foundation translate into, and are defined by A v ~B. = ~A | B. = ~ (B & ~A):

### Table 4

<table>
<thead>
<tr>
<th></th>
<th>Conversion of Asymmetric Functions into Symmetric Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ⇒ B</td>
<td>~ (A &amp; ~B) = ~A v B = A</td>
</tr>
<tr>
<td>A ⇔ B</td>
<td>~ (B &amp; ~A) = A v ~B = ~A</td>
</tr>
<tr>
<td>A acc B</td>
<td>A &amp; ~B = ~ (~A v B) = ~ (A</td>
</tr>
<tr>
<td>A ind B</td>
<td>B &amp; ~A = ~ (A v ~B) = ~ (A</td>
</tr>
</tbody>
</table>

In this way we arrive at a neat picture of the functions of the second quadrant, which reveals its inner rationale while it is self-explanatory:
### Table 5
The Functions and Categories of the Second Quadrant

<table>
<thead>
<tr>
<th>Function</th>
<th>Category</th>
<th>Truth Table</th>
<th>Necessary Logic</th>
<th>Foundation Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \Rightarrow B$</td>
<td>necessary Consequence</td>
<td>TFTT</td>
<td>Entailment</td>
<td>Foundation</td>
</tr>
<tr>
<td>$\sim A \lor B$</td>
<td>Causality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A \Leftarrow B$</td>
<td>necessary Condition</td>
<td>TTFT</td>
<td>Hierarchy</td>
<td></td>
</tr>
<tr>
<td>$A \lor \sim B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sim A \lor B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Function</th>
<th>Category</th>
<th>Truth Table</th>
<th>Randomness</th>
<th>Independence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \acc B$</td>
<td>Randomness</td>
<td>FTFF</td>
<td>Contingency</td>
<td>Fragmentation</td>
</tr>
<tr>
<td>$\sim (A \Rightarrow B)$</td>
<td>Contingency</td>
<td></td>
<td>Indeterminacy</td>
<td>Leap, Rupture</td>
</tr>
<tr>
<td>$\sim (A \Leftarrow B)$</td>
<td>Indeterminacy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A \Leftarrow B$</td>
<td>Independence</td>
<td>FFTF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sim A \land B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sim A \land B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
THE THIRD QUADRANT: IDENTITY AND EQUIVALENCE

It is clear from the preceding discussion of symmetric and asymmetric truth functions that only the context of the quadrant provides the full picture of its functions. Let us now turn to the third quadrant, whose truth functions are quite different and have rarely if ever been treated as a self-explanatory unit. As we shall see, identity and equivalence are converse functions which along with their negations constitute a unit which is no less intricate than the first and second quadrants.

On the whole the third quadrant still waits to be recognized as a self-explanatory unit. On the one hand, equivalence (↔) and its denial, “contravalence” (→<>), or polar opposites (opp), are treated on a par with the truth-functions of the second quadrant and are variously interpreted as "biconditional," "bisubjunction" (Lorenzen 1965: 33), or "mutual implication" (Quine 1982: 25), respectively as "exclusive disjunction" (Lukasiewicz, Bochenski 1964, Quine 1982: 18), or "bisubtraction" (Lorenzen 1965: 30).

By contrast, tautology (TTTT) and contradiction (FFFF) are rarely recognized as truth functions. For the most part, they are treated as synonymous with affirmation and negation (Wittgenstein TPL 5.101; Lukasiewicz; Bochenski 1962) and are subsumed under predicative logic. It is thus not surprising that the internal unity of the quadrant has gone widely unnoticed.

Nobody disputes that identity plays a key role in logic. Identity and non-identity, or difference, are the wellsprings of meaning and of thought in general. In addition, equivalence and polar opposites are the basis of mathematics and measurement. Nobody seems to have considered them components of a common quadrant.

Equivalence and Polar Opposites

At a first glance the third quadrant seems to replicate the symmetry of the first quadrant. However, a closer look on equivalence and opposition shows that the functions of the third quadrant are not merely symmetric, but reflexive and duplicative. Thus "A↔B" does not simply read, "A is equivalent to B, and B is equivalent to A", as this is the case with the functions of the First quadrant. What mutual implication, the so-called biconditional, really
denotes is congruity not only of the two arguments (which explains their reflexivity) but also of their negative complements, i.e., to say “A is congruent with B” implies “~A is congruent with ~B”:

\[
A \leftrightarrow B. = (A \to B. & B \to A) & (\sim A \to \sim B. & \sim B \to \sim A)
\]

\[
= (A \leftrightarrow B) & (\sim A \leftrightarrow \sim B)
\]

\[
= (A \& B) \leftrightarrow (\sim A \& \sim B).
\]

The same principles apply to polar opposites. Not only does A imply non-B, and non-B, A to ensure congruence, but non-A must also imply B, as B must imply non-A:

\[
A \text{ opp } B. = (A \to \sim B. & \sim B \to A) & (\sim A \to B. & B \to \sim A)
\]

\[
= (A \leftrightarrow \sim B) & (B \leftrightarrow \sim A)
\]

\[
= (A \& \sim B) \leftrightarrow (B \& \sim A).
\]

**Tautology and Contradiction**

At the same time, an intricate calculus rules tautology and contradiction which veils their correspondence with equivalence and polar opposites. The puzzle starts when tautology is misconstrued as A=B, and accordingly, contradiction is misconstrued as A ≠ B'. Instead, it is the statement that "A is not identical with B", i.e., "A ≠ B," which is true no matter whether A or B are true or false. It represents the truth function with four positive truth-values, viz. TTTT.

In sum, non-identity is not contradiction. If A and B are different, they are non-identical, but not necessarily contradictory. Rather, to state that A and B are different is to state that they are each identical only with themselves and that identity with anything else is always false. Appearance to the contrary, the schema "A ≠ B" denotes identity and difference, i.e.,

\[
A \neq B. = (A \neq \sim A) & (B \neq \sim B). = (A \mid \sim A) & (B \mid \sim B)
\]

\[
= A=A, \sim A=\sim A, B=B, \sim B=\sim B,
\]

which is always true, TTTT, whatever the values of A or B.

By contrast, contradiction produces four negative truth-values, FFFF. The meaning of contradiction is that it denies that A is not identical with non-~A:

\[
A = B. = \sim (A \neq B). & \sim (A \neq \sim A). & \sim (B \neq \sim B)
\]

\[
= (A = \sim A) & (B = \sim B).
\]

Obviously, tautology and contradiction follow the same rationale as equivalence and polarity: congruence implies reflexivity and duplication. By the same token each function of
the quadrant is convertible to conjunctional and disjunctional form, as the following table brings out:

<table>
<thead>
<tr>
<th>A ≠ B</th>
<th>A ↔ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTTT</td>
<td>TFFT</td>
</tr>
<tr>
<td>A=A, ~A=~A, B=B, ~B=~B.</td>
<td>(A↔B) &amp; (~A ↔ ~B)</td>
</tr>
<tr>
<td>A</td>
<td>~A. &amp; B</td>
</tr>
<tr>
<td>Identity &amp; Difference</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>A = B</td>
</tr>
<tr>
<td></td>
<td>FFFF</td>
</tr>
<tr>
<td>(A = ~A) &amp; (B = ~B)</td>
<td>(A ↔ ~B) &amp; (B ↔ ~A)</td>
</tr>
<tr>
<td>~(A</td>
<td>~A) &amp; ~(B</td>
</tr>
<tr>
<td>Contradiction</td>
<td>Polar Opposites</td>
</tr>
</tbody>
</table>

### The Function of Identity and Difference: Set Theory

Looked at more closely, identification is not only the most basic of all truth functions, because it is the mechanism that generates meaning; it is also the most radical of all functions: Identity absorbs everything that is identical, just as non-identity repulses everything that is not identical\(^\text{10}\).

Moreover, finding out what is different and what is identical in two objects starts the dual process of generalization (by looking for the qualities which are common) and of differentiation (by specifying the qualities which are not common)\(^\text{11}\). It is in this way that sets and subsets are formed and that set theory originates.

It thus becomes clear that far from being limited to sterile tautologies and contradictions, identification and differentiation is the hub of all thought, and that thought harbors all the potential for innovation and creativity. It is thus not true, as Kant has argued, that the predicate is contained in the subject. Invariably the common genus adds information which
reaches beyond its species. Invariably it involves judgment, or as Hegel put it: *die Anstrengung des Begriffs.*

A derivative, abstract “grammar” thus emerges from within ordinary language, which constitutes “pure” logic in the sense that it eliminates all substantive, “satiated” variables by replacing them with “unsaturated,” algebraic variables (Frege). The original life-world in which the individual acts, feels and thinks is thus supplemented by abstractions and generalizations which replace "knowledge by acquaintance" with "knowledge by description" (Russell 1914). Concepts replace percepts, as analytics replaces indexicality.12

Identification and differentiation thus lie at the root of set theory, as the latter lies at the root of analytical theory. While it is true that sets and classes are extensional, there are no sets without descriptions which define them. Invariably extension presupposes intension, i.e.:

\[
\text{INT} \iff \text{EXT.} = \neg \text{INT} \Rightarrow \neg \text{EXT.} \rightarrow "\text{EXT} \rightarrow \text{INT.}"
\]

**The Function of Equivalence and Opposition: Mathematics**

Much has been made of the fact that set theory founds mathematics, including the opposite attempts to reduce mathematics to logic (Frege 1893; Whitehead and Russell 1910) or to reduce logic to mathematics (Boole 1854; E. Schröder 1878). The truth is that mathematics presupposes, but does not reduce to, logic. As Boole (1854) realized, the laws of algebra are not reducible to logic, but he stopped short of acknowledging that mathematics presupposes logic, and in particular the functions of equivalence and opposition (contravalence):

\[
\text{LOG} \iff \text{MATH.} = \neg \text{LOG} \Rightarrow \neg \text{MATH..} \rightarrow \text{MATH} \rightarrow \text{LOG.}
\]

Invariably, mathematics presupposes logic, and in particular equivalence and contravalence, which make mathematics possible. In a similar fashion polar opposites mark extremes such as hot and cold, young and old, and in particular, 0 and 1, which provide the parameters for scaling and measurement. Time and age, size and distance, weight and price, temperature and air pressure, are amenable to measurement by establishing scales. No matter how different the scales, as the differences between miles and kilometers, ounces and grams, etc. illustrate, they are all predicated on polar opposites.
In spite of their indispensability polar opposites, like equivalences, are transfinite. Just as the combination of long vs. short, high vs. low, and narrow vs. broad indicate the dimensions whose Cartesian product founds analytical geometry, i.e.,

\[
\text{Extension} = \text{Length} \times \text{Hight} \times \text{Breadth}
\]

the identification of similar parameters founds analytical social theory\textsuperscript{13}. To give the reader an idea of the logical roots of analytical sociology, it is easy to understand that the following parameters “determine the range that is left open to the facts:”

- **skills** determine the range of the *occupational structure*;
- **property** determines the range of the *property or class structure*;
- **rank** determines the range of the *power structure* of any society, such that

\[
\text{skills} \times \text{property} \times \text{rank} = \text{Social Structure.}
\]

It thus elucidates that identity and difference, equivalence and polar opposites play a crucial role in the constitution of theory than drab notions of tautology and contradiction, or for that matter, Aristotle’s syllogisms suggest
THE FOURTH QUADRANT: APODICTICITY AND CONTINGENCY

Ontological and Logical Necessity

At first sight, our imagination seems at a loss with the fourth quadrant\(^\text{14}\). Its truth-values merely duplicate the tabulation of A or B and thus seem of little cognitive value. The only analogue they call to mind is Mendel’s duality of dominant and recessive genes. As another analogue let us imagine a bus line in which A denotes the schedule, and B, the number of passengers or the weather. Then it is possible to say that “whatever the number of passengers, or whatever the weather, the bus will go as scheduled,” i.e., “Under any circumstances A, no matter whether B or not,”\(^\text{15}\) or simply, “Necessarily A.” i.e.,

$$A \land (B \lor \neg B) = \mathcal{N} A.$$  

Likewise, observing a forest fire B, we may wonder whether it was caused by lightening or arson (A or non-A), i.e., "Whether A or non-A, in any case B," i.e.,

$$(A \lor \neg A) \land B = \mathcal{N} B.$$  

The negative variants to the two examples are easy to imagine. For example, if diabetics must abstain from candy, the rule runs: “Under any circumstances, no candy (~A), whether the patient likes it or not (B or ~B):”

$$\neg A \land (B \lor \neg B) = \mathcal{N} \neg A,$$

or accordingly

$$(A \lor \neg A) \land \neg B = \mathcal{N} \neg B.$$  

All four examples illustrate the dominance of one variable over the other. Accordingly, the truth-value of the function is always that of the argument which dominates, i.e., TTFF and FFTT, respectively, TFFT and FTFT.

In every case the necessity that the statement expresses is judgmental. Necessity and impossibility, possibility and probability are therefore not ontological, but logical categories which imply no causal or teleological determinism. What is commonly categorized as modal logic thus is concerned with modes of judgment and as such replaces – and clarifies - - the classical dispute about truth \textit{a priori} and truth \textit{a posteriori}.
Truth-values as Logical Imperatives

The preceding observations highlight the logical imperatives which inhere in all logical functions, including even the weakest one, viz. disjunction. The equation

\[ A \lor B = \sim A \mid \sim B = \sim (\sim A \land \sim B) = \sim P (\sim A \land \sim B) \]

reveals the hidden imperative which compels us to infer: "Given non~A, necessarily B", respectively, "Given non~B, necessarily A". In every case, the imperative that inheres in every truth function is brought out by its four truth-values:

\[ A \lor B = P AB, A\sim B, \sim AB; \sim P \sim A \land \sim B = TTTF. \]

Likewise, if A excludes B, A and B cannot both be true:

\[ A \mid B = \sim (A&B) = \sim P AB; P A\sim B, \sim AB, \sim A\sim B = FTTT. \]

The same rule holds for conjunction: If A and B form a Cartesian product, neither A nor B can be negative, i.e.,

\[ A \land B = \sim (\sim A \lor \sim B) = N AB; \sim P AB, A\sim B, \sim A\sim B = TFFF. \]

Likewise, if neither A nor B are true, any combination of A and B is impossible:

\[ A \downarrow B = \sim A \land \sim B = \sim P AB, \sim AB, A\sim B; N \sim A\sim B = FFFT. \]

In sum, necessity (N) and impossibility (~P) are the imperatives which determine the four truth functions of the first quadrant:

<table>
<thead>
<tr>
<th>( \sim P \sim A \land \sim B )</th>
<th>( P AB, A\sim B, \sim AB; \sim P \sim A \land \sim B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sim P \sim A \land \sim B )</td>
<td>( N AB; \sim P AB, A\sim B, \sim A\sim B )</td>
</tr>
</tbody>
</table>

Table 7
The First Quadrant in Terms of Modal Logic

A reduction of the second quadrant to modal form is no less instructive. The asymmetry
of the functions finds its expression in the combination of necessity with non-necessity as its open-ended counterpart. Thus entailment combines variant antecedents with a necessary consequent; conversely foundation combines a necessary antecedent with variant consequent, just as randomness and independence bracket necessity of the consequent and the antecedent, respectively.

<table>
<thead>
<tr>
<th>A =&gt; B</th>
<th>A &lt;= B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFTT</td>
<td>TTFT</td>
</tr>
<tr>
<td>= ~P (A &amp; ~B); P AB, ~AB, <del>A</del>B</td>
<td>= ~P (<del>A &amp; B); P AB, A</del>B, <del>A</del>B</td>
</tr>
<tr>
<td>A acc B</td>
<td>A ind B</td>
</tr>
<tr>
<td>FFTF</td>
<td>FFTF</td>
</tr>
<tr>
<td>= N (A &amp; ~B); ~P AB, ~AB, <del>A</del>B</td>
<td>= N (~A &amp; B); <del>P AB, A</del>B, <del>A</del>B</td>
</tr>
</tbody>
</table>

The reduction of the third quadrant to modal form is no less instructive. With unexcelled clarity it brings out the reflexivity of the third quadrant, whose reflexivity allow for no open-ended alternatives.

<table>
<thead>
<tr>
<th>A ≠ B</th>
<th>A ↔ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTTT</td>
<td>TFFT</td>
</tr>
<tr>
<td>A ~A &amp; B ~B</td>
<td>A ~B &amp; ~A B</td>
</tr>
<tr>
<td>= N A=A, <del>A</del>A, B=B, <del>B</del>B;</td>
<td>= N AB, <del>A</del>B;</td>
</tr>
<tr>
<td>A = B</td>
<td>A opp B</td>
</tr>
<tr>
<td>FFFF</td>
<td>FTTF</td>
</tr>
<tr>
<td>~(A ~A &amp; B ~B)</td>
<td>A B &amp; ~A ~B</td>
</tr>
<tr>
<td>= <del>P A=B, A</del>B, ~A=B, <del>A</del>B</td>
<td>= N A~B, ~AB; ~P AB, <del>A</del>B</td>
</tr>
</tbody>
</table>
**The Intricacies of Modal Logic**

Let us now identify the functions of the fourth quadrant. Like those of the first three quadrants, they complement and explain each other. To say that A is necessary is tantamount to saying that non~A is impossible, just as saying that non~A is necessary is tantamount to saying that A is impossible, i.e.,

$$N A. = \sim P \sim A. = N A \mid \sim A,$$

$$N \sim A. = \sim P A. = N \sim A \mid A.$$

By contrast, saying that A is possible is tantamount to saying that non~A is not impossible:

$$P A. = \sim N \sim A. = P A \lor \sim A.$$

$$P \sim A. = \sim N A. = P \sim A \lor A.$$

Obviously, contingency (probability and possibility) is the converse to apodicticity (necessity and impossibility). At the same time an unbridgeable gap separates apodicticity from contingency. The first is strict and constitutes logical truth; the second is indeterminate and constitutes factual truth. In sum, modal logic identifies four modes of judgment, which are convertible only to the extent that they recognize the dualism that separates apodicticity from contingency, and vérités de raison from vérités de fait.

<table>
<thead>
<tr>
<th>Table 10</th>
<th>The Four Modal Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Apodicticity:</strong></td>
<td><strong>Contingency:</strong></td>
</tr>
<tr>
<td><strong>Necessity:</strong></td>
<td><strong>Possibility:</strong></td>
</tr>
<tr>
<td>TTFF</td>
<td>FFTT</td>
</tr>
<tr>
<td>N A. = \sim P \sim A</td>
<td>P B. = \sim N \sim B</td>
</tr>
<tr>
<td><strong>Impossibility:</strong></td>
<td><strong>Probability:</strong></td>
</tr>
<tr>
<td>FFTT</td>
<td>TFFF</td>
</tr>
<tr>
<td>N B. = \sim P \sim B</td>
<td>P A. = \sim N \sim A</td>
</tr>
</tbody>
</table>
Quantification and Modal Logic

A striking homology rules both modal logic and quantification: To say, "all men are mortal" is to say, "there exists not a single man that is not mortal," which is in turn tantamount to saying, "man is necessarily mortal" and "no man is possibly immortal." Obviously, quantifiers and modal logic are homologous.

<table>
<thead>
<tr>
<th>∀x Qx = ~∃x ~Qx</th>
<th>~∀x Qx = ∃x ~Qx</th>
</tr>
</thead>
<tbody>
<tr>
<td>N Q = ~P ~Q</td>
<td>~N Q = P ~Q</td>
</tr>
<tr>
<td>~∀x ~Qx = ~∃x Qx</td>
<td>~∀x ~Qx = ∃x Qx</td>
</tr>
<tr>
<td>N ~Q = ~P Q</td>
<td>~N ~Q = P Q</td>
</tr>
</tbody>
</table>

The gulf that separates apodicticity and contingency thus also separates universal and particular statements (Popper) as well as logical truth and factual truth (Carnap), vérités de raison and vérités de fait (Leibniz). Observation never reaches beyond empirical generalization. It cannot rule out that some x may turn out to be non-Q, i.e.,

∃x Qx = ~∀x ~Qx, = P Qx v ~Qx.

By the same token factual statements are not convertible into universal statements. Conversely, as Popper (1934) has emphasized, the latter are not amenable to verification.

The term "quantificational logic" thus turns out to be a misnomer: To say, “all x without exception are Q” goes beyond quantitative measurement, just as the statement, “some x are Q” does not specify quantity. What the dichotomy of universal and particular quantifiers really addresses is the distinction between two modes of judgment, apodictic and contingent, or in terms of Leibniz, between rational and factual truths.

Husserl's notion of "existential epoche" addresses the same issue, though from a different angle. To say, "No matter whether centaurs (x) exist or not, they are half man and
half horse," is to abstain from factual judgment, i.e.,

\[(\exists x \lor \lnot \exists x) \forall x Qx.\]

Conversely, the factual statement, "Socrates is mortal" implies the suspension of universal judgment, or "categorical epoche": "No matter whether all men are mortal or not, this man Socrates is mortal" translates into

\[(\forall x \lor \lnot \forall x) \exists x Qx.\]

We thus end up with the following table, which provides the analytical formula for Husserl’s notion of epoche: eidetic reduction bars phenomenological reduction and vice versa because the two “bracket” each other. As Husserl always emphasized, his phenomenology is based on reduction, i.e., it transcends empirical description.

<table>
<thead>
<tr>
<th>Universal Apodictic Judgments</th>
<th>Particular Contingent Judgments:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>~Q</td>
</tr>
<tr>
<td>\forall x Qx. = ~\exists x \lnot Qx</td>
<td>\exists x Qx. = ~\forall x \lnot Qx</td>
</tr>
<tr>
<td>\forall x \lnot Qx. = \exists x Qx</td>
<td>\forall x \lnot Qx = \forall x Qx</td>
</tr>
<tr>
<td>\exists x \lnot Qx = \forall x \lnot Qx</td>
<td>\forall x \lnot Qx = \forall x Qx</td>
</tr>
</tbody>
</table>

The same holds for William James’ “radical empiricism,” which is not an ordinary empiricism. Rather, it is synonymous with phenomenological reduction. By contrast, Kant’s notion of transcendentalism turns out to be dual: Looked at more closely, it implies both eidetic and phenomenological reduction: the first, “making science possible,” the second, founding cognition in the -- radically empirical – “transcendental ego.” It is the conundrum of Kant’s transcendentalism that it addresses both types of reduction without being aware that they represent two different kinds of a priori, the first, subjective -- “radically
empirical” -- the second, objective and abstract. It is the latter which makes science possible, as Kant rightly stated, while wrongly giving credit to the transcendental Ego. The latter cannot possibly explain the objective validity of science, which must be credited to Logic.\textsuperscript{16}

Obviously, the gap between universal and factual statements is unbridgeable. Neither can universal statements be induced on grounds of observation, nor can reality be deduced from universals. Universal statements are "transcendental" in the sense that they transcend the here and now, just as, conversely, “radical empiricism” reduces reality to strictly existential statements which are embedded in the here and now – the principle which it shares with historicism. It is in this sense that Bergson and William James were true “transcendentalists” in a sense that Kant was not.

The fourth quadrant is therefore fundamental for the philosophy of science. What Carnap addressed as factual versus logical truth juxtaposes unique, contingent action with abstract, ideal sets, an insight to which Wilhelm Windelband (1883) came close with his dualism of nomothetic and idiographic sciences. The same dualism has found its expression in Roland Robertson’s (1974) pregnant distinction between “actionness” and “systemness” as the axes of sociological theory.

<table>
<thead>
<tr>
<th></th>
<th>Actionness</th>
<th>Systemness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robertson:</td>
<td>contingent</td>
<td>apodictic</td>
</tr>
<tr>
<td>Carnap:</td>
<td>“Factual” Truth</td>
<td>“Logical” Truth</td>
</tr>
<tr>
<td>Leibniz:</td>
<td>\textit{vérités de fait}</td>
<td>\textit{vérités de raison}</td>
</tr>
<tr>
<td>Windelband:</td>
<td>idiographic sciences</td>
<td>nomothetic sciences</td>
</tr>
<tr>
<td>Rescher:</td>
<td>Correspondence Theory</td>
<td>Coherence Theory</td>
</tr>
<tr>
<td>W.James:</td>
<td>metaphysical truth</td>
<td>scientific truth</td>
</tr>
<tr>
<td></td>
<td>\textit{existential a priori}</td>
<td>\textit{transcendental a priori}</td>
</tr>
</tbody>
</table>
Two Types of A priori

The previous table sensitizes us to a fundamental change in our conception of the a priori. As Husserl’s duality of eidetic and phenomenological reduction indicates in tandem with Bergson and William James, the contrast between the two modes of reduction is not that between a priori and a posteriori, as suggested by Kantianism – including Windelband, Rickert and Max Weber -- but between two kinds of a priori, the first, abstract and transcending the here and now; the second, embedded in space and time and thus inherently contingent without being wedded to empiricism, as Husserl’s insistence on phenomenological reduction as opposed to psychology illustrates.

The point is that Husserl’s ingenious notion of eidetic reduction is synonymous with structural reduction, which in turn remarkably fits with Occam’s perspicuous notion of abstract cognition as opposed to intuitive cognition – the first, “transcending” the here and now, the second, embedded in it. It is for this reason that Husserl’s “transcendental” reduction turns out to be a misnomer, for what his phenomenological reduction has to do is exactly reduce empirical reality to something that William James addresses as “radical empiricism” and “metaphysical cognition.” It comes therefore as no surprise that both Husserl and William James sympathized with Dilthey and Bergson. Phenomenological reduction fits historicism once it is understood that the latter implies its own kind of a priori, which is no longer wedded to structure and set theory, but to sedimentation in one’s past, both individually and collectively: Everything that exists is rooted in its past, an insight which culminates in Nietzsche’s notion of amor fati.

What Husserl called eidetic and phenomenological reduction thus denotes structural, systematic reduction, respectively, existential, idiographic (biographic) reduction, which replaces abstract systems by concrete, unduplicated Wholes, or in terms of Leibniz, Monads.

We have thus to give William James and Bergson credit for having preceded Husserl, we also have to mention the three eminent thinkers that preceded them: Occam (1323), Leibniz (1714), and Schopenhauer (1818). At the same time it elucidates that for all its seeming simplicity, empiricism, including empirical sociology, turns out to be a duplicitous mixture
of both kinds of a priori without recognizing them.

THE GENERAL STRUCTURE OF LOGIC

As the preceding analyses demonstrate, the sixteen truth functions become fully transparent only if they are arranged in the four quadrants which reveal their specific rationale, viz. commutativity, irreversibility, reflexivity, and recidivity, or reduction. Most important, the widespread conception of a strictly formal, “mathematical” logic (logistics) is complemented by the notion of a substantive logic, which is clothed in language as an instrument not only of communication, but of cognition. What has generally touted as “linguistic turn” thus turns out to be the turn to semiology in the sense of C.S. Peirce and C.I. Lewis (1932) and to "transcendental logic" in the sense of Husserl (1929). The myth of logic as a sort of Platonic side-heaven that miraculously underlies reality thus evaporates together with its positivist counterpart, which reduces logic to a mere technology regardless of cognitive implications.17

As the following tables demonstrate, the sixteen truth functions produce miracles of consistency, variability and perspicuity once they are arranged by quadrants:

<table>
<thead>
<tr>
<th>I.</th>
<th>II.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A v B</td>
<td>A ⊨ B</td>
</tr>
<tr>
<td>T T</td>
<td>F T</td>
</tr>
<tr>
<td>T F</td>
<td>T T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A ⊨ B</th>
<th>A &amp; B</th>
<th>A ind B</th>
<th>A acc B</th>
</tr>
</thead>
<tbody>
<tr>
<td>F F</td>
<td>T F</td>
<td>F F</td>
<td>F F</td>
</tr>
<tr>
<td>F T</td>
<td>F F</td>
<td>T F</td>
<td>F T</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The two axes which marshal the four quadrants thus reveal the rationale, as it were, the "deep structure" of logic, viz. symmetry vs. asymmetry in the vertical, and description vs. construction in the horizontal axis:

### Table 15
The Second Two Quadrants: Identity and Modality

<table>
<thead>
<tr>
<th>III.</th>
<th>IV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ≠ B</td>
<td>A opp B</td>
</tr>
<tr>
<td>T T</td>
<td>F T</td>
</tr>
<tr>
<td>T T</td>
<td>T F</td>
</tr>
<tr>
<td>N A</td>
<td>P B</td>
</tr>
<tr>
<td>T T</td>
<td>T F</td>
</tr>
<tr>
<td>T T</td>
<td>F F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A = B</th>
<th>A ⇔ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>F F</td>
<td>T F</td>
</tr>
<tr>
<td>F F</td>
<td>F T</td>
</tr>
</tbody>
</table>

The two axes which marshal the four quadrants thus reveal the rationale, as it were, the "deep structure" of logic, viz. symmetry vs. asymmetry in the vertical, and description vs. construction in the horizontal axis:

### Table 16
The Two Axes of Logic

<table>
<thead>
<tr>
<th>Description (correspondence)</th>
<th>Symmetry:</th>
<th>Asymmetry:</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. commutative</td>
<td>II. unidirectional</td>
<td></td>
</tr>
<tr>
<td>Coordination (atomic sets)</td>
<td>Subordination (conditionals)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Determination (coherence)</th>
<th>Modes of Judgment (quantification)</th>
</tr>
</thead>
<tbody>
<tr>
<td>III. reflexive</td>
<td>IV. reductive</td>
</tr>
<tr>
<td>Identification (predication)</td>
<td></td>
</tr>
</tbody>
</table>
Logic Pure and Applied

In contrast to "pure," formal logic applied logic is intrinsically dual and in this sense, synthetic. Its rigorous character notwithstanding, it combines formal calculi with substantive arguments. It is therefore subject to falsification. By the same token, logical truth (L-truth) is subject to continual revision both of its variables and its functions. The first focuses on piecemeal revision (Popper 1934); the second, on paradigm change (Kuhn 1970).

Kant's notion of synthesis a priori thus finds a new interpretation: It is the synthesis of substantive content with logical grammar, i.e., of substantive reference (Bedeutung) with syntactical meaning (Sinn). Taken by itself, logic would be confined to the ivory tower.

Table 17
Logic Pure and Applied

<table>
<thead>
<tr>
<th>pure logic:</th>
<th>applied logic:</th>
</tr>
</thead>
<tbody>
<tr>
<td>p v q</td>
<td>denotes</td>
</tr>
<tr>
<td>p ↓ q</td>
<td>denotes</td>
</tr>
<tr>
<td>p ⏜ q</td>
<td>denotes</td>
</tr>
<tr>
<td>p &amp; q</td>
<td>denotes</td>
</tr>
<tr>
<td>p → q</td>
<td>denotes</td>
</tr>
<tr>
<td>p acc q</td>
<td>denotes</td>
</tr>
<tr>
<td>p ← q</td>
<td>denotes</td>
</tr>
<tr>
<td>p ind q</td>
<td>denotes</td>
</tr>
<tr>
<td>p ≠ q</td>
<td>denotes</td>
</tr>
<tr>
<td>p = q</td>
<td>denotes</td>
</tr>
<tr>
<td>p ↔ q</td>
<td>denotes</td>
</tr>
<tr>
<td>p opp q</td>
<td>denotes</td>
</tr>
<tr>
<td>N p</td>
<td>denotes</td>
</tr>
<tr>
<td>N ~p</td>
<td>denotes</td>
</tr>
<tr>
<td>P q</td>
<td>denotes</td>
</tr>
<tr>
<td>P ~q</td>
<td>denotes</td>
</tr>
</tbody>
</table>

adjunction, inventory, enumeration
joint denial, rejection, dismissal
exclusion, incompatibility
complementarity, inseparability, intersection
sufficient reason, necessary consequence, causality
randomness, accidentalness, chance, openness
necessary condition, foundation, emergent hierarchy
discontinuity, rupture, leaps, fragmentation
uniqueness, singularity, difference
contradiction, invalidity, error
equivalence, synonyms, substitution
contravalence, antonyms, polar opposites
necessity, apodicticity, certitude
impossibility, elimination of error
possibility, novelty, creativity
probability, conjecture, measurement
We thus end up with a clear distinction between pure and applied logic: "Pure," algebraic logic is confined to truth-possibilities (Wahrheitsmöglichkeiten) and truth-conditions (Wahrheitsgründe). These turn into truth-values when saturated variables are substituted for the unsaturated ones. By the same token, formal logic is intrinsically analytic, just as applied logic, i.e., language, is intrinsically synthetic. In any case the conjunctional normal form reveals the truth-conditions for each of the 16 truth-functions. The following table demonstrates the unity of logic and by the same token, its completeness and self-sufficiency:

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Conjunctural Normal Form</th>
<th>Truth-Conditions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTTT</td>
<td>pvq</td>
<td>(~p &amp; ~q)</td>
</tr>
<tr>
<td>FFFT</td>
<td>p↓q</td>
<td>~p &amp; ~q</td>
</tr>
<tr>
<td>FTFT</td>
<td>p</td>
<td>q</td>
</tr>
<tr>
<td>TFFF</td>
<td>p &amp; q</td>
<td>p &amp; q</td>
</tr>
<tr>
<td>FTPF</td>
<td>p→q</td>
<td>~(p &amp; ~q)</td>
</tr>
<tr>
<td>FFFT</td>
<td>p acc q</td>
<td>p &amp; ~q</td>
</tr>
<tr>
<td>TFTF</td>
<td>p←q</td>
<td>~(~p &amp; q)</td>
</tr>
<tr>
<td>FFFT</td>
<td>p ind q</td>
<td>~p &amp; q</td>
</tr>
<tr>
<td>TTTT</td>
<td>p≠q</td>
<td>p&amp;p.&amp;q&amp;q&amp;q</td>
</tr>
<tr>
<td>FFFF</td>
<td>p=q</td>
<td>p&amp;~p.&amp;q&amp;q&amp;q</td>
</tr>
<tr>
<td>TFFT</td>
<td>p↔q</td>
<td>p&amp;q.&amp;~p&amp;~q</td>
</tr>
<tr>
<td>TFFT</td>
<td>p opp q</td>
<td>p&amp;~q.&amp;q&amp;~p</td>
</tr>
<tr>
<td>TFFT</td>
<td>N p</td>
<td>p &amp; (q v ~q)</td>
</tr>
<tr>
<td>FFFT</td>
<td>N ~p</td>
<td>~p &amp; (q v ~q)</td>
</tr>
<tr>
<td>TFFT</td>
<td>N q</td>
<td>(p v ~p) &amp; q</td>
</tr>
<tr>
<td>FTFT</td>
<td>N ~q</td>
<td>(p v ~p) &amp; ~q</td>
</tr>
</tbody>
</table>
It is therefore erroneous to equate strict science with mathematics or with formalization. In every case, it is logic that produces apodictic truth, which is not limited to the natural sciences. The point is that each of the sixteen truth-functions can be converted into any other \textit{salva veritate}, i.e., so long as its truth-values remain unaltered, as De Morgan's Law anticipated:

\[
A \land B = \neg(\neg A \lor \neg B) = \neg A \downarrow \neg B = FFT .
\]

Clearly, human thought is predicated on logic, to which it owes its rigor and precision even at a pre-linguistic stage.

\textbf{Logic as a Whole}

Contrary to the belief that logic is strictly technical and bare of quality, applied logic reveals the specific qualities of each of the four quadrants and of each of the sixteen truth functions. Just as truth is not separable from meaning, applied logic is synonymous with cognition.

<table>
<thead>
<tr>
<th>1st Quadrant: Coordination</th>
<th>2nd Quadrant: Subordination</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Indifference vs. Nexus</strong></td>
<td><strong>Condition vs. Consequence</strong></td>
</tr>
<tr>
<td>Alternation</td>
<td>Foundation</td>
</tr>
<tr>
<td>Exclusion</td>
<td>Entailment</td>
</tr>
<tr>
<td>Rejection</td>
<td>Independence</td>
</tr>
<tr>
<td>Conjunction</td>
<td>Randomness</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3rd Quadrant: Predication</th>
<th>4th Quadrant: Modality</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identity vs. Computation</strong></td>
<td><strong>Apodicticity vs. Contingency</strong></td>
</tr>
<tr>
<td>Difference</td>
<td>Necessity</td>
</tr>
<tr>
<td>Equivalence</td>
<td>Possibility</td>
</tr>
<tr>
<td>Contradiction</td>
<td>Impossibility</td>
</tr>
<tr>
<td>Polar Opposites</td>
<td>Probability</td>
</tr>
</tbody>
</table>
The Dual Roots of Logic and Pragmatism

Our emphasis that all necessity and possibility is logical rather than ontological does not prevent us from asking about the ontological foundations of logic. The answer to this question is as surprising as it is instructive: Just as Husserl distinguished between two modes of reduction, our knowledge stems from two quite different sources: *eidetic*, or structural reduction, which is analytic, and *transcendental* or *phenomenological* reduction, which is synthetic. The first reduces the perceived reality – not reality *per se* – to homogeneous, but abstract sets. By contrast, *transcendental* reduction synthesizes (unifies) the Person’s experiences into a meaningful whole, which constitutes his *existence*. The latter thus represents a work of art of the Person’s own making which is in constant flux because it changes with every subsequent decision.

Husserl is thus attending to a type of psychology which radically changes Kant’s notion of transcendentalism. Like Fichte and Schelling, Schopenhauer and Nietzsche, Husserl no longer asks how (objective) science is possible, but rather, how the “pure” (transcendental) Ego works when it is unencumbered by external constraint. Husserl “pure” Ego works like a Leibnizian *Monad*. He thus ends up with (subjective, real, ever changing) *Noeseis* and (objective, ideal, abstract) *Noemata* as the primal elements of cognition and of action.

Husserl thus deserves credit for opening a new approach to philosophy which is in striking parallel with Bergson and William James, with the important difference that Husserl remained focused on logic, while James focused on Praxis as the primal givens which for him, like for Bergson, overlapped with Ethics and Religion. One has to widen the picture to include German historicism and French existentialism to obtain the full picture of James’ “radical empiricism.” The latter starts a philosophy which is no longer focused on abstract *noemata* – and that is, on logic -- but on real, spontaneous, contingent acts which eschew logic, being predicated on Schopenhauerian elementary Will, Bergsonian *élan vital* and Nietzschean “Will to Power,” with Leibnizian *Monads* as the famed, if fictitious precursors.
What was not clear to Kant and Fichte is the fundamental difference between impersonal sets and truth-functions, on the one side, and contingent, spontaneous acts, on the other side: The first transcending space and time; the second irrevocably embedded in time and space, and even where they anticipate the future or memorize the past enchained with the present.

Whether he deals with Perikles or Ceasar, Benjamin Franklin or Thomas Paine, the historian describes their life in the present tense. For the same reason the existentialist describes action as churning the future out of the present. History and existentialism thus sensitize us to the fact that action generates time.

By the same token action resists generalization. The latter is the hallmark of logic and of science. By contrast, as epics, the drama and the novel illustrate, action is predicated on biography. Every biography thus constitutes a self-sufficient Monad which follows its own rationale, which James identifies as “radical empiricism,” and which is identical with his notion of pragmatism.

There exists therefore a vast area which is different from logic and is not reducible to it, viz. action, which follows its own rationale: the never ending, unrelenting quest for subjective meaning, or meaningfulness. The latter permeates the work of William James, who at the same time expressly recognizes abstract concepts as indispensable instruments to cope with the external environment. Logic is therefore ubiquitous in all our dealings with our surroundings, starting with the first flint used as a knife and the first cries to express joy or to warn of danger, with artifacts as its elementary instruments.

Husserl’s two modes of reduction thus provide the keys to reality: to analyze and to explain the external world, on the one hand, and to “understand” and empathize action, on the other hand; the first, culminating in logic and science, the second, culminating in pragmatism, ethics and religion as the Sinngebung des Sinnlosen.

What Husserl disguised as eidetic and transcendental or phenomenological reduction thus turns out to address structural, respectively, existential reduction, the first known since Plato and Pythagoras, the second anticipated by Leibniz, but ultimately secured only by
Dilthey and Husserl, Bergson, William James and Sartre under so many different names.

**Logic, Language and Mathematics**

Much of the pioneering work of modern logic has focused on pure (mathematical, algebraic, formal) logic, including advances in modal logic and set theory, but relatively few efforts have been made to draw a sharp distinction between pure and applied logic and to create a distinct theory of pragmatic, effective logic in terms of Peirce and Lorenzen. The result has been a tendency to consider logic an esoteric discipline like higher mathematics or to reduce it to mere convention in the guise of language games.

It seems therefore timely to supplement formal logic with a substantive, cognitive interpretation which reaches beyond mere tautologies. Like tools and signs, whose rationale lies in their use, the functions of logic must be seen as cognitive instruments which serve to come to grips with reality. As Lorenzen and the intuitionists (e.g., Brouwer 1983) have argued, there exists a primitive, inchoate logic of the senses, a sort of protologic, which Quine (1995, p.23), too, recognizes in birds, apes and infants, and which may be the reason for his naturalistic interpretation of logic.

In any case, a sharp distinction must be made between language, logic and mathematics. While they are all three products of the intellect, they serve different functions: Language is man's most eminent instrument of communication -- and as such, the well-spring of meaning -- and mathematics, his most eminent instrument of measurement and computation. By contrast, logic is man's most eminent instrument of thought, which lies at the bottom of both language and mathematics, as it were, as its deep structure.

While language creates icons, indices and symbols as the necessary carriers of meaning (Peirce 1955), what lies at the heart of logic is the combination of four truth-values, or more precisely, truth-possibilities -- Wittgenstein's (TLP 5.105) ingenious discovery. The latter underlie all thought no matter what its content and thus supports the idea of Einheitswissenschaft (Schlick 1928) without reducing it to physics or mathematics. If anything establishes the unity of science it is logic.
**The Constitution of Theory**

To elucidate the cognitive function of logic, one problem must be solved in the first place: How reality translates into logic. The task is as much to elucidate the logical foundations of cognition as to elucidate the cognitive foundations of logic. Both complement each other. This goal is achieved in three steps:

(1) To overcome the plethora of empirical observations, all particular, factual statements must be reduced to universal, apodictic statements. In contrast to ordinary, everyday language, which is based on indexicality, logical truths are the product of abstraction and generalization, which transcends the here and now. Logical truth is therefore radically different from factual truth.¹⁹

(2) The next step is to reduce our language to simple “atomic sentences,” or "canonical notation" (Quine 1960: 226ff), i.e., to elementary propositions, which are stripped down to two variables and whose relationship is regimented by one of the sixteen truth functions. The latter constitute the syntactic meaning, or the logical syntax (Carnap), of human thought.

(3) The two preceding steps provide the building blocks for analytics. However, to create theory one more step is required which is missing in most textbooks: systematicity, or in Kantian terms, architectonics. As Kant (1781, A 474) put it:

Human reason is by nature architectonic. That is to say it regards all our knowledge as belonging to a possible system and therefore allows only such principles as do not at any rate make it impossible for any knowledge that we attain to combine into a system with other knowledge.

In a special chapter on "The Architectonic of Pure Reason" Kant (1781, A 832) elaborates: By an architectonic I understand the art of constructing systems. As systematic unity is what first raises ordinary knowledge to the rank of science, that is, makes a system out of a mere aggregate of knowledge; architectonic is the doctrine of the scientific in our knowledge...

In accordance with reason's legislative prescriptions our diverse modes of knowledge must not be permitted to be a mere rhapsody, but must form a system...

It is the unity of the system which makes it possible for us to determine from our
knowledge of the other parts whether any part be missing, or to prevent any arbitrary addition...

The whole is thus an organized unity and not an aggregate. It may grow from within, but not by external addition. It is thus like an animal body the growth of which is not by addition of a new member, but by rendering each member... stronger and more effective for its purposes.

While we do not share Kant's belief in *a priori* knowledge independent of experience, we owe him the cue for defining strict science: systematicity. The latter coincides with his notion of pure reason (*Vernunft*) as the source of apodictic truth,\textsuperscript{20} which in turn fits with the dichotomy of *cognition ex datis* and *cognition ex principiis* (A 836).\textsuperscript{21}

While it makes sense to interpret Kant's notion of pure reason as pure, formal logic, the latter fails to establish any substantive link with reality. Ironically, the solution of the puzzle lies in Kant's notion of architectonics. In his own terms, systems are wholes whose parts are intricately connected and explain the whole as much as the whole explains the parts.\textsuperscript{22} Theory is therefore not founded in observed regularities, but in the validity of its architectonics. Its stringency notwithstanding, it allows for, and indeed invites, free *Ideenvariation* so long as the truth-values remain unaltered.

It is thus the necessary connexion of forms -- Plato's notion of *sympleke ton eidon* -- that constitutes rational truth. What starts with the infants' perception of identities and differences thus culminates in Aristotle's hierarchy of matter, plants, animals and humans, in Newton's law of gravity, in Planck's quantum theory and in Einstein's theory of relativity. They all derive the validity of their theses not from observed regularities, but from the compound of logical consistency, parsimony and completeness\textsuperscript{23} that lies at the bottom of Kant's notion of architectonics.

**Ontological Relativity and Logical Determinacy**

The notion of logical regimentation also sheds light on "ontological relativity" (Quine 1969). As Althusser (1968), following Lenin, has argued, historical reality is overdetermined in the sense that real (natural and historical) events are determined by a plurality of contingent factors. By contrast, logical regimentation produces ideal constructs which, for
the sake of Denkökonomie (Mach) and simplicity (Bunge), bracket all intervening factors. Inevitably, parsimony and precision are bought at the cost of completeness.

Conversely, ontological completeness is bought at the price of analytical determinacy. Looked at more closely ideal constructs and "things-in-themselves," analysis and intuition are polar opposites which are mutually exclusive: Logical determinacy implies ontological relativity; conversely, observational plenitude implies logical indeterminacy. Which truth function to choose is for the observer to decide. In the last analysis, "ontological relativity" is a misnomer which attributes the indeterminacy of reference to the reality it refers to (Quine 1990: 50).

For an illustration, let us consider a situation which is characterized by the interaction of two variables, a and b, in an in determinate setting K, e.g.,

\[(a + b) \times (a + b) + K = aa + 2ab + bb + K.\]

Then differentiation toward a \( \partial a \) results in

\[\partial a (aa + 2ab + bb + K) = 2a + 2b,\]

i.e., all combinations not containing "a" are bracketed from our view. For all its precision the differential is underdetermined compared with the plenitude of the original equation. This becomes all the more clear if we decide to restore the original plenitude by way of integral calculus:

\[\int a (2a + 2b) = aa + 2ab + b + c + \ldots + K\ldots\]

allows for an indefinite variety of additional variables (c, d, etc.) and constants (K), leaving everything undetermined that is not related to a. Invariably, specific determinacy is bought at the price of general indeterminacy, and vice versa, i.e., analyticity is inversely related to indexicality. The notion of ontological pluralism thus sensitizes us to the fact that the choice of truth functions is in the eyes of the observer rather than in the "thing itself." It is the fundamental fallacy of realism to believe otherwise.

**Logic and Epistemology**

A glance at the strict tautologies of formal logic highlights the difference between pure and applied, formal and, for lack of a better term, "material," or in terms of Husserl (1929),
"transcendental" logic. It is in this sense that logic is dual. On the one hand, applied logic owes its precision, validity and objectivity to formal logic. On the other hand, applied logic establishes the link between monothetic form and polythetic perception, and that is, between syntax and semantics.

For all the rigor it owes to "logistics," science gains immensely in perspicuity, scope and versatility by combining formal rigor with perception, as the leap from strict implication to causality and foundation, randomness and independence and from universality and particularity to necessity and impossibility, possibility and probability demonstrates.

By the same token strict science is no longer limited to mathematics and the natural sciences. Applied logic covers anything from physics to society, ethics and culture. In sum, the rift between natural and "human sciences" is bridged as soon as the right functions combine with the right variables. Accordingly, the way to sociology as a strict, analytical science leads neither through mathematics nor through formalization, but through set theory.

June 26, 2003
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Notes

1. By the same token, formal logic transmutes into transcendental logic.

2. Quine admits: "No doubt, this result seems strange," but adds that it would not be any less strange to construe it as false.

3. Patrick Suppes (1957: 15) and W.V. Quine (1983) address the same point when they speak of "tautological" and "logical" implication, respectively.

4. Sartre's work on *Being and Nothingness* in tandem with Bachelard's notion of *rupture* and discontinuity may be considered the philosophical equivalent to the theories of Einstein and Planck.

5. Neither Wittgenstein (TLP 5.101) nor Bochenski (1959) nor Carnap (1954) or Quine (1982; 1986) developed the notion of quadrants. On the other hand, the notion of converse, joint denial and dual implies the notion both of an elementary zero-form and of a quadrant, with De Morgan's (1842) law as the earliest manifestation. A notable exception is Paul Lorenzen (1965: 34f), who addresses the 1st and 2nd quadrants.

6. Thus Quine (1990: 14) divides logic into truth functions, quantification, and identity. In a similar fashion Lorenzen (1965) distinguishes between logic of functors, logic of quantors, and logic of identity.

7. We thus take the Fregian and Russellian position that mathematics presupposes logic. While we often speak of formal logic as "mathematical" logic insofar as it remains limited to pure schemata, what we really mean is "algebraic" logic. In no case is logic reducible to mathematics.

8. By the same token the third quadrant has nothing to do with quantification, which constitutes the fourth quadrant. Indeed Lorenzen (1965: 75) criticizes Quine for subsuming the logic of quantors under predicate logic. He also observes that predicate logic is often subdivided into lower and higher predicate logic, which correspond to quantification and set theory, respectively.

9. It is noteworthy that Wittgestein (TLP 5,101) did not commit this error. He interpreted tautology as "p→p.&.q→q," and contradiction as "p&~p.&.q&~q".

10. In terms of arithmetic, identity is defined as 1x1 = 1, and 1:1 = 1.

11. One is reminded of Aristotle's formula *Omnis definitio fit per genus proximum et differentiam specificam*.

12. It is thus obvious that the so-called linguistic turn, if it is to be taken seriously, is really a logical turn.

13. For an application of this idea to social structure see Sorokin 1964 and Mueller 1989 on
Analytical Sociology.

14 Note that Wittgenstein's (1922, p.75) computation of the 16 truth functions does not recognize the four modal functions as such. The same holds for Bochenski and Menne (1962, p.31), who speak of "Präpendenz," "Postpendenz," and their denials, "Pränonpendenz," "Postnonpendenz."

16 In other words, Kant never realized that his transcendental philosophy was limited to *eidetic* reduction and did not really deal with the “transcendental Ego.”

17 The attacks on logistics, e.g., by Freytag-Löringhoff (1961) centered on the failure to link logic with cognition and language, a shortcoming which the "linguistic turn" tried to mend at the expense of formal logic.

18 Peirce's (1955, ch.7) trichotomy remarkably parallels Freud's trichotomy of Id, Ego and Superego. It recognizes protologic in the form of icons and thus concurs with Lévi-Strauss' (1968) notion of *The Savage Mind*.

19 In terms of Husserl (1900) it is the function of logic to transform polythetic perceptions into monothetic truth.

20 Kant thus attributed to an imaginary "pure reason" what rightly belongs to logic. What Kant could not know is that his *syntheses apriori* reduce to the sixteen truth functions.

21 Accordingly Kant (1781, A 371) considers himself an empirical realist as well as a transcendental idealist depending on which kind of cognition he refers to.

22 For the same reason Hegel has a point when he notes that all our proofs are circular.

23 By the same token Hegel had a point when he noted that all our proofs are ultimately circular.