Practice Problems
Applied Calculus I

1. Find the derivatives $dy/dx$ where $y$ is a function of $x$, either explicitly or implicitly. You do not need to simplify your answer.
   a) $y = -7x^6 + 4x^3 + \frac{x}{2} - 1$
   b) $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$
   c) $y = (3x^5 + 4x + 2)^2(x^2 + 1)$
   d) $y = \frac{\ln x}{x^3 - 9}$
   e) $y = (4x^5 - 3x^2 - 1)^{10}$
   f) $y = x \ln(x^3 + x^2 + x)$
   g) $y = e^{\sqrt{x+1}} + xe^x$
   h) $y = \ln(x^3 - 1)^2 + \ln 10 + [\ln(x^3 - 1)]^2$
   i) $xy = y^2 + y$
   j) $e^{x^2} - x \ln y = e$
   k) $y = x^3 - 2$
   l) $y = 10^x$
   m) $y = 2^x$
   n) $x^2 + y^2 = 25$
   o) $y = \log_{10} x$

2. a) Compute the derivative of $f(x) = -2x^2 + 11$ using the definition of the derivative.
   b) Compute the derivative of $f(x) = 1/(x + 1)$ using the definition of the derivative. What is $f''(2)$?

3. Compute the following limits if they exist. If a limit does not exist, state this. Justify your answer.
   a) $\lim_{x \to 3} (x - 4)^2$
   b) $\lim_{x \to -3} \frac{x^2 - 5x + 6}{x^2 - 9}$
   c) $\lim_{x \to 2} \frac{x + 2}{x - 2}$
   d) $\lim_{x \to -1} \frac{|x + 1|}{x + 1}$
   e) $\lim_{x \to -\infty} \frac{-x^{11} + 7x^{10} + 3x}{10 - 30x^{11}}$

4. a) Sketch the graph of $f(x) = \begin{cases} x + 3, & \text{if } 2 < x \\ x^2 + 1, & \text{if } -2 < x < 2 \\ -x & \text{if } x \leq -2. \end{cases}$
   b) What is the domain of $f(x)$.
   c) Is $f(x)$ continuous for all $x$? If not, what are the discontinuities of $f(x)$?

5. Find equations for the horizontal asymptotes of the given function $g(x) = \begin{cases} 5x^2 + 4x + 3 & \text{if } z \geq 0 \\ -z^2 + 1 & \text{if } z < 0 \\ 3x + 1 & \text{if } z \leq 0 \end{cases}$ if any exist.
6. Find equations for the vertical asymptotes of the given functions
   a) \( f(x) = \frac{x^2 + 4}{x^2 - 4} \)
   b) \( f(x) = \frac{x^2 + 2x + 1}{x^2 - 1} \).

7. The picture below is the graph of the function \( g(x) \).

   ![Graph of g(x)](image)

   a) List all the \( x \)-intercepts, if there are any.
   b) What is the \( y \)-intercept?
   c) What is the domain of \( g(x) \)?
   d) Determine equations for the vertical asymptotes.
   e) Find \( \lim_{x \to -3} g(x) \) if it exists.
   f) Find \( \lim_{x \to 0} g(x) \) and \( \lim_{x \to -\infty} g(x) \).
   g) Determine equations for the horizontal asymptotes if there are any.
   h) What is \( g(-2) \)?
   i) Find \( \lim_{x \to -2} g(x) \) if it exists. Find \( \lim_{x \to 2} g(x) \) if it exists. Find \( \lim_{x \to -2} g(x) \) if it exists.
   j) Is \( g(x) \) continuous at \( x = -2 \)? Is \( g(x) \) continuous at \( x = 3 \)? Justify your answer.
   k) Determine the intervals on which \( g(x) \) is increasing and decreasing.
   l) What are the relative maxima?
   m) What are the relative minima?
   n) Determine the intervals on which \( g(x) \) is concave up and concave down.
   o) If there are any points of inflection, list them.

8. a) Find the intervals where \( f(x) = (x + 2)^{4/5} \) is increasing and decreasing and identify the relative extrema if they exist.
   b) Find the intervals where \( y = xe^x \) is increasing and decreasing and identify the relative extrema if they exist.
   c) Find the intervals where \( f(x) = -x^3 + 2x - 6 \) is increasing and decreasing and identify the relative extrema if they exist.
9. a) Find the intervals where \( f(x) = x^4 - 4x^2 + 6 \) is concave up and concave down, and identify the points of inflection if they exist.

b) Find the intervals where \( y = \frac{1-x}{x} \) is concave up and concave down, and identify the points of inflection if they exist.

10. a) Find the absolute extrema for the function \( f(x) = 2 \ln x - 2x \) on \([1/e, 3e]\).

b) Find the absolute extrema for the function \( f(x) = x^2 e^x \) on \([-1, 1]\).

11. A chartered bus requires a minimum of 20 persons. If 20 persons sign up, the cost is $20 per person. For each additional person the cost is reduced by $0.50. Find the number of passengers that maximizes the revenue. What is the maximum revenue? What is the marginal revenue when there are 35 passengers?

12. Compute the following:
   a) \( e^{\ln \sqrt{1000}} \)
   b) \( 7^{\log_7 1} \)
   c) \( \log_{10} \sqrt{10} \)
   d) \( \log_2 \frac{1}{e} \)

13. Find all values of \( x \) which satisfy the following:
   a) \( e^{x^2 - 25} = 0 \)
   b) \( e^{4x} = 10 \)
   c) \( 20 = 4e^{x/2} \)
   d) \( \ln|x| = 1 \).

14. An undergraduate business student with $10,000 to invest wishes to have $20,000 available after 2 years with which to launch a small business. What annual rate of interest, compounded continuously, will produce this yield?

15. Compute the following indefinite integrals. (i.e., find the general antiderivatives.)
   a) \( \int (10x^3 - x^2 + 5x + 5) \, dx \)
   b) \( \int \frac{1}{\sqrt{x}} \, dx \)
   c) \( \int (x + 2)e^{3x^2 - 12x} \, dx \)
   d) \( \int \frac{1}{x(\ln x)^3} \, dx \)
   e) \( \int \frac{-x^3 + x + 1}{x} \, dx \)
   f) \( \int e^{\sqrt{x}} \, dx \)
   g) \( \int \frac{x}{x^2 + 5} \, dx \)
16. Compute the following definite integrals
   a) \( \int_0^1 x(x^2 + 1)\,dx \)
   b) \( \int_1^2 \frac{1}{x+5}\,dx \)
   c) \( \int_0^1 xe^x\,dx \)
   d) \( \int_1^2 \frac{1}{3x+2}\,dx \)
   e) \( \int_0^1 e^{x-2}\,dx \)

17. Find the area of the region bounded by the graphs of the functions \( f(x) = x^2 - 4 \) and \( g(x) = x - 2 \).

18. A manufacturer finds that its marginal cost at production level \( x \) in the production of a certain product is
   \[ MC = C'(x) = 800 - 11x + 4x^2. \]
   Find the total cost function \( C(x) \) if \( C(0) = 1991 \) dollars.

19. A manufacturer of dishwashers can produce up to 150 dishwashers per week. Sales experience indicates that the manufacturer can sell \( x \) dishwashers per week at price \( p = 3x = 600 \) dollars. Production records show that the cost of producing \( x \) dishwashers per week is \( C(x) = 500 + 150x + 0.5x^2 \).
   Find the weekly revenue function.
   Find the weekly profit function.
   Find the marginal cost and marginal revenue functions.
   Find the weekly production level for which profit is a maximum.
   Find the maximum profit.
   Find the profit and marginal profit if 100 dishwashers per week are sold.

20. When a spherical balloon is being blown up, the volume is increasing at the rate of 100 cubic centimeters per second. How fast is the radius increasing when the radius is 5 centimeters?

21. If the half-life of an element is one year, how long does it take for 20% of a quantity of the element to disappear?

22. a) Find an equation for the tangent line to the graph of the function \( y = \frac{1}{x-1} \) at the point \( 2, 1 \).
   b) Find an equation for the tangent line to the graph of the function \( y = \sqrt{x+2} \) at the point \( 0, \sqrt{2} \)

23. An open box is to be made from a square sheet of cardboard by cutting out squares of equal size from each of the four corners and bending up flaps. The sheet of cardboard measures 20 cm on each side. Find the dimensions of the box of maximum volume that can be made in this way.

24. Graph \( f(x) = x^3 - x^\frac{3}{2} \). Determine the regions where \( f(x) \) is increasing/decreasing, concave up/concave down. Find the critical numbers, relative extrema, inflection points, \( x \) and \( y \)-intercepts, and horizontal and vertical asymptotes whenever they exist.

25. Graph \( f(x) = \frac{x^2 - 1}{x + 2} \). Determine the regions where \( f(x) \) is increasing/decreasing, concave up/concave down. Find the critical numbers, relative extrema, inflection points, \( x \) and \( y \)-intercepts, and vertical and horizontal asymptotes whenever they exist.