INNOVATION AND ECONOMIC DYNAMICS

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Contents:

Summary:

This chapter provides a survey of recent economics research on innovation and dynamic change. First, an overview of different models of innovation is provided. The focus here is on models of product innovation, rather than process innovation. The former kind accounts for the bulk of innovative activities. Two types of innovation are considered: horizontal (expansion in the variety of goods) and vertical (improvements in the quality of goods). Second, applications of the models are provided. The focus of the applications is on policy-oriented issues; for example, the relationship between market size and innovation, which has implications for policies on economic integration, and the role of subsidies to research and development. Another policy issue is patent protection versus open innovation in which researchers do not seek patent rights but instead freely share inventions and discoveries. The analysis of this issue has implications for how best to reward innovation and for how best to disseminate it.

1. Introduction

In this chapter, recent economics research on the determinants of innovation is surveyed. A key contribution of the economics literature has been to show, among other things, that innovation can respond to economic incentives. Indeed the literature emphasizes that innovation is an intentional activity driven by profit-maximizing entrepreneurs interacting with consumers (or users) who display a preference for new and better quality goods. A better understanding of the economic influences on innovation is important in formulating public policy towards science and technology.

This guide to the research proceeds in two steps: first, a survey of the leading dynamic models of innovation is provided. These models have served as workhorses for much of the recent economic studies on innovation. The models blend research from economic growth theory and industrial organization. Second, these models are applied to address public policy issues; for example, what is the role of market size (or scale)? How do subsidies to research and development (R&D) and stronger patent systems affect innovative activity? Is 'open innovation' an alternative to proprietary, profit-seeking modes of innovation? The chapter illustrates how the dynamic models of innovation can be used to gain some insights on the role of technology policy.

The canonical models surveyed here share some common features. For example, innovation rates are higher if consumers are more patient and willing to save for the future, firms have market power, research workers are more productive, and the economy has more resources. They also share the feature that the private market need not deliver the socially optimal rate of innovation. The private market could under-invest or over-invest in R&D depending on the type of innovation, whether horizontal (expansion in the variety of goods) or vertical (improvements in the quality of goods).

Furthermore, the models predict a 'scale effect', meaning that larger economies have higher rates of innovation. Recent evidence seems to cast doubt on this prediction. Hence the canonical models can be modified so as to eliminate the scale effect. One way is to assume that R&D becomes more difficult to conduct as the level of innovation rises. The equilibrium rate of innovation then depends not on the level of resources but on the growth rate of resources (like labor). Another way to eliminate the scale effect is to assume that larger economies are associated with more industrial sectors so that R&D resources must be spread more thinly across the economy. This has the effect of making the equilibrium rate of innovation depend on, not the scale of R&D, but on the share of R&D inputs in total resources.

Technology policies can be used to influence the long run equilibrium level and/or rate of innovation. But there is weak theoretical consensus on the effects of R&D subsidies. It is plausible for subsidies to have beneficial as well as adverse effects on innovative activities. On patent policies, the consensus seems to be that they stimulate R&D but up to a point. If patent protection is too strong, innovation can be adversely affected due to excessive market power and due to the higher cost of conducting R&D (because of higher licensing and royalty fees). An alternative approach, therefore, to innovation is for individual researchers to forgo patent rights and engage in open innovation (such as 'open source software' or 'open biotech'). In some cases, though, open innovation can be complementary to proprietary (patent-seeking) innovation, particularly if it makes fundamental research tools accessible to all researchers. However, the overall impact of open innovation on the economy-wide rate of innovation is ambiguous due to the possibility that some open innovation may displace for-profit innovation.

R&D subsidies, patents, and open innovation are among the leading technology policy issues in the recent literature – and are suitable issues to analyze using the kinds of dynamic models surveyed in this chapter. Nonetheless, they do not exhaust the full range of influences on innovative activities. Thus the chapter provides some follow-up literature for the interested reader.

The chapter is organized as follows: Section 2 contains a review of the basic models of innovation. Section 3 addresses the issue of scale effects – that is, of whether larger economies have a higher rate of long run innovation. Section 4 discusses the impact of R&D subsidies on innovation, Section 5 the impact of patent rights on innovation, and Section 6 the potential role of open innovation, where innovators do not assert patent claims. Section 7 provides concluding thoughts.

Before proceeding, address the scope of this chapter should be addressed. First, recent work is reviewed, starting about in the early 1990s. For earlier surveys of innovation, the interested reader could consult Kamien and Schwartz (1982) and Tirole (1988, chapter 10). Second, theoretical and conceptual analyses are covered, rather than empirical studies. Third, the focus is primarily on the relationship between research and development (R&D) and innovation, and the chapter does not address other determinants of innovation such as human capital, trade policy, financing, and so forth. Fourth, the focus is on product innovations where innovation results in new or improved products, rather than process innovations where innovations result in new or improved methods of production. Most patented inventions tend to be product innovations, whereas process inventions are often protected by trade secrecy laws.

2. Canonical Models

Innovation here has two dimensions: horizontal and vertical. Horizontal innovation involves the creation of new varieties of goods, while vertical innovation improves the quality of existing goods. The goods in question can be final consumption goods or intermediate inputs into production. Thus there are four cases to consider: (i) innovation in the variety of final goods; (ii) innovation in the variety of intermediate inputs; (iii) innovation in the quality of final goods; and (iv) innovation in the quality of intermediate inputs. Table 1 shows a classification of the different types of innovation and the studies that belong under the different categories of innovation. A glossary provides a list of key symbols used in this chapter.

Table 1. Examples of Innovation Studies by Type

Before going into the four cases, the common elements are discussed. Three key actors are modeled: the consumer, producer, and innovator. The market for innovation consists of consumers that demand goods and producers that supply them. The market provides a value for innovation.

- Demand Side

$$U = \int_0^\infty e^{-\rho t} \ln D_t dt \qquad (1)$$

This equation shows the lifetime utility households derive from the stream of consumption from time zero to infinity, where D is an index of consumption and ρ the time preference rate. For presentational purposes, a logarithmic utility function is assumed throughout this chapter, even though different kinds of functions are used in the literature. The purpose of adopting a common functional form for utility is to make economical use of notation and to minimize technical detail.

The specification for D depends on whether the goods are homogenous or differentiated. If the latter, the differentiation can be in the variety of goods or in the different quality levels of goods:

$$D = \left[\int_0^n c(j)^\alpha \, dj \right]^{\frac{1}{\alpha}} \qquad 0 < \alpha < 1 \qquad \text{Case of horizontal differentiation} \qquad (1a)$$

or

$$D = \exp\left(\int_0^1 \ln q(j) c(j) \, dj \right) \qquad \text{Case of vertical differentiation} \qquad (1b)$$

where c denotes consumption of the *j*-th good, n the measure of variety, and q an index of quality. In (1a), the smaller the parameter α the more substitutable the goods are.

- Production

Producers can use capital (denoted by x) and labor (denoted by L_Y) to produce goods. The capital inputs can be subject to innovation, either in the variety of capital inputs or in the quality of inputs.

$$Y = \left(\int_0^n x(j)^\beta \, dj \right) L_Y^{1-\beta} \qquad \text{Case of horizontal differentiation} \qquad (2a)$$

or

$$Y = (q\, x^\beta)\, L_Y^{1-\beta} \qquad \text{Case of vertical differentiation} \qquad (2b)$$

The parameter β measures the output elasticity (or sensitivity) of output to capital. In some of the models below (particularly where innovation occurs only at the final goods level), capital is not used to produce output. In that situation, it will be assumed that one unit of labor is required to produce one unit of output (and hence $\beta = 0$).

- Research and Development (R&D)

Innovation requires resources, namely labor denoted by L_R.

$$\dot{n} = \frac{L_R}{A} n \qquad \text{Case of horizontal differentiation} \qquad (3a)$$

or

$$\phi = \phi(A, L_R) = \frac{\phi(L_R)}{A} \qquad \text{Case of vertical differentiation} \qquad (3b)$$

where ϕ is the probability of successfully developing an improved quality good. In (3a), the instantaneous change in n is a function of the level of n. This indicates the presence of knowledge spillovers. Past innovation (as embodied in the level of n) facilitates further innovation. Equation (3b) also implicitly assumes knowledge spillovers in that the probability of success does not depend on cumulative research effort. The current state of the art captures what the researcher needs to know in order build a better product. In both (3a) and (3b), the parameter A measures productivity. The lower A is, the more efficient researchers are at innovation.

- Resource Constraint

$$L = L_R + L_Y \qquad (4)$$

L is the total endowment of labor to be allocated between research and production.

- Market Clearing

Total output produced, Y, is divided between consumption and investment.

$$Y = C + I \qquad (5)$$

where investment, I, is used to augment the stock of capital, K. Hence $I = \dot{K}$ and $K = \int_0^n x(j)dj$.

Aggregate consumption C is allocated among different varieties of goods:

$$C = \int_0^n p(j)c(j)\,dj \qquad (6)$$

where $n = 1$ in the case of quality ladders models (where a continuum of industries exist along the unit interval).

- Consumer Utility Maximization

The utility maximization decision can be broken down into two steps. First, consumers make a static decision in which they optimally allocate their spending, C, across different goods at a given point in time. Assuming symmetry of goods (i.e. $c = c(j)$ for all j)

$$c = \frac{C}{pn} \qquad \text{Static Maximization}$$

Second, consumers make a dynamic decision in which they optimally determine the path of their spending, C, over time. This is done by maximizing Eq. (1) subject to a lifetime budget constraint (at time 0). The solution to this dynamic problem is referred to as the *Euler equation*:

$$\frac{\dot{C}}{C} = r - \rho \qquad \text{Dynamic Maximization} \qquad (7)$$

where r is the interest rate. Total consumption grows (or falls) according to whether the market rate of interest, r, is greater (or less) than the personal rate of interest, ρ. For an introduction to methods of dynamic optimization, the interested reader is referred to Klein (2002).

- Firm Value Maximization

The following pertains to the link between innovation and production. Innovation yields a "blueprint" or a design for a new good or an improved good. Producers pay a fixed cost of F to innovate (or to buy the blueprint from others). The producers are then given a patent right to be the exclusive supplier of this new or improved good. The value of the firm (and value of the innovation) equals the presented discounted value of profits associated with selling this good:

$$V = \int_0^\infty e^{-rt} e^{-\phi t} \pi_t \, dt = \frac{\pi}{r + \phi} \qquad (8a)$$

Note that the discount factor includes the risk ϕ that an innovation by another firm will destroy the stream of profits. In horizontal R&D models, no technological displacement or obsolescence occurs so that $\phi = 0$.

If $V < F$, innovation is not profitable, while if $V > F$, innovators will enter the market to innovate. In the long run, equilibrium requires the following condition to hold.

$$V = F \qquad \text{Free-Entry Condition} \qquad (8b)$$

The profitability of innovation (given by 8a), the free-entry condition (given by 8b), and the overall resource constraint (given by 4) interact to determine the overall equilibrium rate of innovation. The other ingredients, such as consumer tastes, innovation productivity, and so forth, are embedded or incorporated into these conditions.

With these building blocks, four cases are next analyzed. For each case, the model is solved in order to derive the equilibrium innovation rate for the private market. The equation for the innovation rate provides insight into the underlying determinants of innovation. Then an examination is made of whether the private market rate of innovation is socially optimal. In each case, the private market does not necessarily generate the socially optimal rate of innovation. The rest of the chapter then addresses technology policies that can influence private innovation.

2.1. Case 1: Horizontal Differentiation of Final Goods

This section is based on Grossman and Helpman (1991a, Chapter 3). For this case, we use Eqs. (1), (1a), (3a), (4), (6), (8a), and (8b). Let $g = \frac{\dot{n}}{n}$ be the rate of innovation. To ultimately derive the equilibrium g, we start by figuring out firms' profits and thus the value of innovation. It is useful to start with the consumer's demand for goods of different varieties. This is what for-profit firms look to in order to determine whether there is a market for innovative goods. Static maximization results in the following demands for individual goods:

$$c(j) = \left(\frac{p(j)^{\frac{1}{1-\alpha}}}{\int_0^n p(j)^{\frac{\alpha}{\alpha-1}} dj} \right) C \qquad (9)$$

Equation (9) can be incorporated into the profits of the producer of the *j*-th good:

$$\pi(j) = (p(j) - w)c(j) \qquad (10)$$

where w denotes wage. Maximizing profits yields:

$$p = \frac{w}{\alpha} \qquad (11)$$

Due to patent rights and the fact that goods are not perfect substitutes, the producer gets to charge a price that is a markup above the wage. Under competitive conditions, the price would equal marginal cost, which is the wage. Incorporating this markup pricing rule in (10) yields a level of profits equal to:

$$\pi = \frac{(1-\alpha)C}{n} \qquad (12)$$

In these models, symmetry (i.e. where $c(j) = c$ for all j) naturally arises since w and α are the same for all j. Thus symmetry is not imposed but a result of various assumptions. This expression will be used in (8a) and (8b) to characterize the value of innovation.

Our next step is to incorporate the resource constraint and determine the allocation of labor. We use Eq. (6) to find that $L_Y = nc = \frac{C}{p}$. This is because one unit of labor is required to produce one unit of output. From (3a), we can find the quantity of labor in research: $L_R = \frac{\dot{n}}{n}A$. Using the resource constraint (4) we can find that:

$$L = gA + \frac{C}{p} \qquad (4)'$$

Returning to the valuation conditions (8a, b), it can be seen from (3a) that the number of workers per variety is $\frac{L_R}{\dot{n}} = \frac{A}{n}$. Hence the cost of innovation is wA/n. This is the fixed cost, F, of creating a new variety. The free-entry condition is therefore $V = F = \frac{wA}{n}$. Given constant w and A, if we time-differentiate this free-entry condition, we see that the rate of change in varieties is linked to the rate of change in the firm's value:

$$\frac{\dot{V}}{V} + \frac{\dot{n}}{n} = \frac{\dot{V}}{V} + g = 0.$$

But upon time-differentiating Eq. (8a) – after imposing $\phi = 0$ since there is no obsolescence or displacement of goods in this model – we get:

$$\frac{\dot{V} + \pi}{V} = r \qquad (13)$$

Equation (13) resembles the standard ("no arbitrage") asset pricing equation. The left hand side of it is the rate of return to owning a share in the firm. The numerator consists of the capital gains plus profits and the denominator the equity value of the firm. The right side is the interest rate that could be earned in an alternative asset (like bonds). If the rates of return to the assets (bonds and equities) were not equal, there would be opportunities for arbitrage.

The equilibrium innovation rate takes into account the evolution of firm value, the opportunity cost of investing in an innovating firm, and the resource constraint. Thus, using Eqs. (4)', (12), the Euler equation in steady state (where $r = \rho$ in this model), and the fact that $\frac{\dot{V}}{V} = -g$, Eq. (13) becomes:

$$g = (1 - \alpha)\frac{L}{A} - \alpha\rho \qquad (14)$$

This is the market equilibrium rate. The question is whether this is a socially optimal innovation rate.

The following is an outline of the steps for calculating the optimal innovation rate (i.e. growth rate of varieties). In Eq. (1a), by symmetry, $D = n^{\frac{1}{\alpha}} c = n^{\left(\frac{1-\alpha}{\alpha}\right)} Y$. Note that all labor is used to produce quantities of output. Thus, $Y = L_Y$. In turn, $L_Y = \int_0^n c(j)\,dj = nc$. Next, substitute the above expression for D into Eq. (1), the lifetime utility function. The goal then is to maximize (1) subject to the resource constraint $L = L_R + L_Y = Ag + Y$. The present-value Hamiltonian is

$$\mathrm{H} = e^{-\rho t}\left[\left(\frac{1-\alpha}{\alpha}\right)\ln n_t + \ln Y_t\right] + \mu\left(\frac{L-Y}{A}\right) \qquad (15)$$

The necessary conditions are $\frac{\partial H}{\partial Y} = 0$ and $\frac{\partial H}{\partial n} = -\dot{\mu}$, where μ is the co-state variable or shadow price of an extra variety. The solution to this problem is:

$$g^* = \frac{L}{A} - \left(\frac{\alpha}{1-\alpha}\right)\rho \qquad (16)$$

A transversality condition also needs to be satisfied, namely the condition that the present discounted value of the total shadow value of varieties, μn, converge to zero as time goes to infinity. Comparing (14) and (16) shows that, from society's point of view, the private sector's rate of innovation is slower than optimal. The principal reason is that innovation creates positive externalities. Each innovator contributes to the stock of knowledge which raises the productivity of future researchers but the innovator does not receive compensation from future researchers. Thus the innovator does not take into account the benefit to future researchers when choosing a level of R&D effort.

2.2 Case 2: Horizontal Differentiation of Intermediate Inputs

This section is based on Romer (1990). For this case, we use Eqs. (1), (2a), (3a), (4), (5), (7), (8a), and (8b). Again the rate of innovation is measured as $g = \frac{\dot{n}}{n}$, except that n is a measure of the variety of intermediate inputs into production. Consider three sectors on the production side. The R&D sector produces blueprints, which are purchased by the intermediate goods sector. Each blueprint provides a design for an intermediate input. The intermediate goods sector then rents these inputs to producers in the final goods sector. The producers in the final goods use these capital goods to help produce final output. The intermediate goods sector is characterized by monopolistic competition: there are many firms that produce close but not perfectly substitutable capital goods. Each firm has market power due to product differentiation and due to patent protection which allows it to be the exclusive provider of an input of a specific design.

Let us start at the top with the manufacturing sector. Differentiating Eq. (2a) with respect to x gives us the final goods sector's demand schedule for the input (i.e. the marginal productivity of x). Let p_x be the price the final producers are willing to pay for the use of an additional x:

$$p_x(x(j)) = \frac{\partial Y}{\partial x(j)} = \beta\, x(j)^{\beta-1}\, L_Y^{\;1-\beta} \tag{17}$$

The *j*-th monopolistically competitive firm in the intermediate goods sector faces the demand curve above. Its profits are:

$$\pi(j) = [\,p_x(x(j)) - r\,]x(j)$$

where r is the rental price of capital. Profit maximization entails $\frac{\partial \pi(j)}{\partial x(j)} = 0$, the solution to which yields a markup pricing formula and profits of:

$$p_x(x(j)) = \frac{r}{\beta} \tag{18}$$

$$\pi(j) = (1-\beta)\; p_x(x(j))\, x(j) \tag{19}$$

respectively. For the privilege of selling the *j*-th intermediate input exclusively the intermediate goods producer purchases a design from the research sector. By the free-entry condition:

$$\mathrm{V} = \frac{\pi(j)}{r} = \mathrm{F}$$

To determine what F is, we turn to the R&D sector. Profits here are:

$$\pi_R = F\dot{n} - wL_R \tag{20}$$

where F is the price of the design which the research sector sells to an intermediate goods producer and w the wages paid to R&D labor. To maximize profits, the research sector hires that quantity of labor at which $\frac{\partial \pi_R}{\partial L_R} = 0$. Solving this yields $F = \frac{wA}{n}$.

Thus the free-entry condition ($V = F$) can be written as:

$$V = \frac{(1-\beta)\, p_x x}{r} = \frac{wA}{n} \qquad (21)$$

We also need to recognize that labor has competing uses. It can work in the manufacturing and research sectors. Thus the following condition determines the equilibrium allocation of labor:

$$\frac{\partial Y}{\partial L_Y} = w = \frac{nF}{A} \qquad (22)$$

The left hand side is the marginal productivity of labor in the manufacturing sector and the right hand the same in the research sector. Labor would be paid more in the sector in which it has a higher marginal product. So, if there were differences in wages, labor would migrate to whichever sector offered the higher wage. But through migration, the marginal productivities would equalize because the marginal productivity of labor falls as more labor is available and rises as less labor is available.

The equilibrium innovation rate satisfies the no-arbitrage condition, the free entry condition, and the resource constraint. Thus using (17), (21), (22), the Euler equation $r = g + \rho$, and the fact that $x = \frac{K}{n}$, $L_Y = L - L_R$, and $L_R = gA$ (by (3a)) gives us the equilibrium growth rate of new innovative varieties in the private sector:

$$g = \frac{\beta\left(\frac{L}{A}\right) - \rho}{1 + \beta} \qquad (23)$$

Again is this innovation rate in the private market socially optimal? A social planner would maximize (1) subject to the constraints (3a) for research capital accumulation and (5) for physical capital accumulation, where the control variable is C and the state variables are n and K (research capital and physical capital respectively). By the method of Hamiltonians, the solution is:

$$g^* = \left(\frac{L}{A}\right) - \rho \qquad (24)$$

Comparing (23) and (24) again shows that the private rate of innovation is slower than optimal. The reason again is that innovation generates positive externalities. Future innovators get to enjoy a larger stock of knowledge. Moreover, the increased stock of knowledge increases the marginal productivity of inputs in the final goods sector. Neither of these benefits is appropriated by the current innovators.

2.3. Case 3: Vertical Differentiation of Final Goods

This section is based on Grossman and Helpman (1991b). For this case, we use Eqs. (1), (1b), (3b), (4), (6), (7), (8a), and (8b). Here innovation results in improvements in the quality of existing goods. The quality level of a good evolves as follows: $q^J(j) = \lambda q^{J\text{-}1}(j)$, for $j \in [0,1]$, where $\lambda > 1$ represents size of the quality jump and J indexes the version of the good. If we normalize $q^0(j) = 1$, then $q^J(j) = \lambda^J$.

The innovation rate is measured by ϕ, the probability of success (see (3b)). Our goal here is to derive the equilibrium value of ϕ. The value depends on the private sector's allocation of resources to R&D, and that in turn depends on the profitability of innovation. The market demand for quality goods comes from the consumer sector, so that is where we start.

Recall that static consumer utility maximization leads to $c(j) = \dfrac{C}{p(j)}$ for $j \in [0,1]$ and dynamic utility maximization leads to the Euler equation given by (7). Again, symmetry is a feature of this model so that $c = C/p$.

On the producer side, the quality of the leader's good is λ times that of the follower's. The leader in each product line prices its good strategically such that $p = \lambda w$ for each *j*. If $p > \lambda w$, consumers value the new quality version but not at that price so that the quantity demanded is zero. If $p < \lambda w$, the firm is not maximizing profits. Hence, λ is the markup. Note that this assumes that the innovation is not drastic. Otherwise, $p = w/\alpha$ since the leader is not constrained by the follower's quality level. Incorporating the consumer's demand into the firm's profit gives:

$$\pi = (p - w)c = (1 - \frac{1}{\lambda})C \qquad (25)$$

so that the firm's value is $V = \dfrac{\pi}{r+\phi}$.

On the R&D side, only the followers conduct research because the model is set up so that the marginal gain from having a 2 step quality advantage is smaller than the marginal gain from acquiring a 1-step advantage. Followers therefore have greater incentives to undertake R&D. To conduct R&D, the innovator requires ϕA units of labor to achieve a success rate probability of ϕ. Note that the probability of success itself, ϕ, for simplicity is assumed to be independent of the quantity of R&D workers employed. Given this labor requirement, the fixed cost of R&D (of innovating a better quality) is $F = w\phi A$. Against this cost is the expected benefit of R&D. With probability ϕ, the innovator will earn a market value of V and with $(1\text{-}\phi)$ the innovator will earn nothing. Hence the expected benefit of R&D is $\phi V + (1\text{-}\phi)0 = \phi V$.

Thus the free entry condition is $\phi V = F$, or $V = wA$. Substituting profits π from (25) into $V = \dfrac{\pi}{r+\phi}$ and the result in turn into the free-entry condition (using $r = \rho$) gives:

$$wA = \frac{(\lambda-1)C}{\lambda(\rho+\phi)} \tag{26}$$

Using the resource constraint $L = L_R + L_Y = \phi A + \frac{C}{p} = \phi A + \frac{C}{\lambda w}$ to eliminate C gives the private market equilibrium rate of innovation:

$$\phi = (1 - \frac{1}{\lambda})\frac{L}{A} - \frac{\rho}{\lambda} \tag{27}$$

This rate ϕ satisfies the resource constraint and the no-arbitrage and free-entry conditions.

Again, the issue is whether this equilibrium rate of innovation is socially optimal. From (1b),

$$\ln D = \int_0^1 \ln \lambda^J \, c(j)\, dj = J \ln \lambda + \ln Y \qquad \text{(1b)'}$$

Again, $Y = L_Y = \frac{C}{p}$ since all output (of all product lines) is produced by labor (with an input-output coefficient of one). In (1b)', J is the number of quality changes hitherto. More specifically, $J_t = \int_0^t \phi_s \, ds$ is the number of quality jumps during an interval of time t, given the arrival rates ϕ's. Substituting (1b)' into (1), and maximizing social welfare function (1) subject to the resource constraint $(L = \phi A + Y)$ will yield the socially optimal rate of innovation. The Hamiltonian:

$$H = e^{-\rho t}(J_t \ln \lambda + \ln Y_t) + \mu \dot{J}_t$$

where $\dot{J}_t = \phi_t = \left(\frac{L - Y_t}{A}\right)$.

With Y as the control variable and J the state variable, the solution to this problem is

$$\phi^* = \frac{L}{A} - \frac{\rho}{\ln \lambda} \tag{28}$$

This time comparing (27) and (28) would reveal that the private rate of innovation ϕ could be higher or lower than the socially optimal rate ϕ^*, depending on the value of the quality jumps λ. For either small values of λ or large values of λ, the private rate is above the optimal ($\phi > \phi^*$). For intermediate values of λ, the private rate is below the optimal ($\phi < \phi^*$). The intuition is that while private innovation creates spillovers for future research and increases in consumer surplus (i.e. greater quality per price of goods), innovation in the context of vertical innovation destroys the profits of existing producers of quality goods. In other words, quality innovations impose a “business stealing effect”. The profits destroyed also impose a cost on consumers who own shares of existing firms and derive dividend income. Moreover, innovators who enter the market and destroy existing profits anticipate that they may one day be displaced but due to time discounting, they weigh their losses less than the profits of others that they destroy. It is because of the presence of these opposing spillover effects that the private rate of innovation may be too high or too low from a social welfare point of view. If quality jumps are small in size, the consumer surplus effect and knowledge spillovers are small in size. Hence it is more likely that the business stealing effect will dominate and cause the private innovation rate to be too rapid. If quality jumps are very large in size, the business stealing effects will be larger in magnitude. Hence again private rates of innovation will be too fast. Thus it is for intermediate sizes of quality jumps that the private rate of innovation will be slower than socially optimal.

2.4. Case 4: Vertical Differentiation of Intermediate Inputs

This section is based on Aghion and Howitt (1992) and Aghion and Howitt (1998, Chapter 2). For this case, we use Eqs. (2b), (3b), (4), (5), (7), (8a), and (8b). We turn now to changes in the quality of an intermediate input (and assume final goods are homogeneous). Let $L_Y = 1$. A fixed quantity of factors works in the manufacturing sector. Thus (2b) simply becomes:

$$Y_t = q_t\, x_t^{\beta} \qquad \text{(2b)'}$$

where t denotes innovation interval, not time, and $q^t = \lambda^t$. Using (2b)', the derivative of output with respect to x gives the final good sector's demand for the input. Let p_x again denote the price of the input.

$$p_x = \frac{dY}{dx} = q\, x^{\beta - 1} \qquad (29)$$

To avoid cluttering up the notation, the presentation below omits the subscript t until it is necessary to distinguish transitions between innovation intervals.

The intermediate sector is characterized by a monopoly, unlike in case 2 which described a monopolistically competitive sector producing varieties of inputs. Here there is just one intermediate input that undergoes quality change and there is a sequence of monopolists that appear, each providing a better quality level than the previous producer.

Thus the monopoly producer of an intermediate input lasts until the next innovation occurs which displaces him from the market. The inputs are produced using labor; for example, one unit of input x requires one unit of labor. Thus $x = L_x$, where the latter denotes the quantity of labor employed in the production of intermediate inputs. The monopoly producer's flow of profits is $\pi_x = (p_x - w)x$, where w again denotes wages.

Using (29) in the expression for profits, we can obtain the first-order conditions for profit maximization, which again yields a markup pricing formula: $p_x = \dfrac{w}{\beta}$ and profits of $\pi_x = \left(\dfrac{1-\beta}{\beta}\right) wx$. This is similar to case 2 except that here labor, instead of rental capital, is hired to produce x. The value of a firm reflects the risk that the firm could lose the stream of profits upon innovation by a follower; hence $V = \frac{\pi_x}{r+\phi}$.

The R&D sector also hires labor to work on uncertain research projects. The probability of a successful breakthrough is given by (3b) earlier. Rearranging (3b) gives $A\phi(A, L_R) = \phi(L_R)$, which resembles that of case 3, except that here the success rate, ϕ, is a function of the quantity of labor employed. It is convenient to assume a linear functional form for ϕ; namely, $\phi(L_R) = L_R$. Thus the probability of success is:

$$\phi(A, L_R) = \frac{L_R}{A}$$

The research sectors' profits are

$$\pi_{RD} = \phi(A,\, L_R)V - wL_R = \frac{L_R}{A}V - wL_R \qquad (30)$$

Maximizing these profits lead to the first-order condition whereby $V = wA$. This is again the free-entry condition.

To derive the equilibrium R&D, substitute the expression for profits into the expression for the value of the firm, and the resulting expression into the free-entry condition. Then, use the resource constraint $L = L_R + L_x = L_R + x$ to eliminate x. Rearranging gives:

$$L_R = (1\text{-}\beta)L\text{-}\beta A\rho \qquad (31)$$

This shows the equilibrium amount of labor allocated to innovative activities by the private sector.

Again does the private market allocate the optimal quantity of labor to research and development? To find out, we examine the social planner's problem. The social welfare function is:

$$U = \int_0^\infty e^{-\rho t} \underbrace{\sum_{s=0}^{\infty} f(s,t)\,\lambda^s\,(L - L_R)^\beta}_{Y}\, dt \qquad (32)$$

where $x = L\text{-}L_R$ was substituted into the production function. Here $f(s,t)$ is the probability of s jumps at time t, given that innovation follows a Poisson process. That is,

$$f(s,t) = \frac{(\phi t)^s\, e^{-\phi t}}{s!}$$

where $\phi = L_R / A$. Solving (32) yields:

$$U = \frac{(L - L_R)^{\beta}}{\rho - \frac{L_R}{A}(\lambda - 1)}$$

Setting $\frac{dU}{dL_R} = 0$ enables us to derive the socially optimal allocation of workers to research and development:

$$L_R * = \left(\frac{1}{1-\beta}\right)L - \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{\lambda-1}\right)\rho A \qquad (33)$$

Comparing (31) and (33) shows that the private market's allocation of resources to R&D may be higher or lower than the socially desired allocation. Again, the core reason has to do with the creative destruction aspects of vertical innovation. While this type of innovation also generates positive knowledge spillovers which the innovators do not fully appropriate, quality innovations generate business stealing effects. The private research firm ignores these losses to previous firms when choosing its innovation effort.

To summarize, the four cases tend to show that equilibrium innovation or R&D is higher if the economy has more resources (L is higher), innovators are more productive (lower A) and if the consumers are more patient (lower rates of time preference, ρ, and thus have a higher propensity to save for the future). Larger markups, which yield greater profits, are also an inducement to innovation. However, the equilibrium rates of innovation tend to be lower than socially optimal for horizontal innovations, but can be higher or lower for vertical innovations due to the fact innovation destroys the profits of existing firms. The private market need not always under-invest in horizontal innovation. Benassy (1998) shows a case where private innovation could exceed the socially optimal amount when the returns to specialization in varieties are sufficiently low. With this basic framework, we turn to some applications of the models.

3. Scale Effects

An issue of interest is what happens to the rate of innovation as the scale or size of the economy increases. Scale or market size could be measured by aggregate income, population, or resources (for example, labor supply). In the four cases considered above, the answer is that a "positive" effect occurs: an increase in scale is associated with a higher rate of long run innovation.

Recent evidence appears to challenge that prediction. For example, Jones (1995) shows that the number of scientists and engineers has increased in the U.S.A. during the past half century, yet growth rates of total factor productivity (TFP) exhibit no upward trend. Thus, if L represents human capital or skilled labor, the evidence does not seem to show that an increase in L is associated with an increase in g.

Of course, a number of empirical challenges need to be addressed. For one thing, the theory predicts a "long run" association between g and L, not short run. Thus empirically it is necessary to control for business cycles or waves. It is also necessary to control for other factors that may drive changes in both g and L or have effects that offset any impact that scale has had on rates of innovation.

Structural or regime shifts may also occur so that in certain periods or epochs, a positive, negative, or neutral relationship is observed. Thus both theoretical and empirical frameworks need to take into account the possibility of shifting relationships, rather than assume relationships that hold for all time periods and circumstances. Verspagen (2004) has noted that neoclassical growth theory has tended to be a bit mechanistic at times, failing to incorporate evolutionary aspects of technical change:

Translating theoretical frameworks into empirical analyses also requires care. For example, how is scale defined and measured? Is it population, GDP, skilled labor, or a weighted aggregate of skilled resources including capital? How is the growth rate defined? In the above canonical models, balanced growth is assumed, so that the rates of growth of output, consumption, and knowledge capital are the same. In a world where economies are not on a balanced growth path, or where economies are in transition and from time-to-time perturbed by shocks, the variables identified in theoretical models do not all grow at the same rate. In such circumstances, changes in scale would have different effects on the out-of-equilibrium behavior of output and innovation. Thus, empirically, which measure of the growth rate would have the strongest or weakest relationship, if any, to scale? This adds to why dynamic adjustments need to be properly specified in empirical work so as to capture the intent of theoretical models.

Nonetheless, the consensus in the recent literature appears to be that there is a lack of consistent empirical support for a positive relationship between g and L. Thus much attention in this literature (in the mid to late 1990s) has been to modify the canonical models above so that they would not exhibit "scale effects" – to derive models where the equilibrium g is invariant to changes in L or any other measure of economic scale.

Several approaches to purging scale effects are possible, two of the most common ones will be outlined. The first is to assume that R&D difficulty increases as the level of innovation increases. The intuition is that researchers tend to discover the most obvious ideas first so that as the stock of knowledge and innovative activity increase, new innovations become more difficult to generate. The second approach is to assume product proliferation as the scale of an economy increases. The intuition is that the proliferation of products helps to exhaust the rents from increased market size. R&D workers are spread more thinly among products (or the sectors that produce them) so that increases in the supply of R&D workers do not translate into higher rates of innovation. We discuss these in turn.

3.1. R&D Difficulty Approach

This approach is motivated by the fact that a key source of scale effects in the canonical models is the strong intertemporal spillovers: the absolute productivity of an innovator is positively influenced by the stock of past innovations. Thus, in Jones's (1995) modified model, for example, the absolute productivity of R&D workers is assumed to be constant or not dependent on past innovations. Due to increasing R&D difficulty, more and more resources need to be invested in order to sustain a given percentage growth rate of knowledge. But as resources are finite, eventually the growth rate would fall or equal the rate of growth of resources.

To see this, recall Eq. (3a) where $\frac{\dot{n}}{n} = \frac{L_R}{A}$. Here we modify it so that

$$\frac{\dot{n}}{n} = \frac{1}{A}\left(\frac{L_R}{Z}\right) \qquad (34)$$

where Z is an index of R&D difficulty. Jones (1995) adopts this approach for case 2 (variety growth model) and Segerstrom (1998) adopts it for case 3 (quality ladders model). This index Z increases with R&D, so that over time technological opportunities are diminishing. Researchers pick the low-hanging fruit first and work their way up, figuratively speaking. Employing Eq. (34) modifies the equilibrium innovation growth rate. It is no longer a function of the *level* of labor but the *growth rate* of labor:

$$g = g\left(\frac{\dot{L}}{L}\right),$$

where the first derivative is positive, i.e. $g' > 0$.

The problem, though, is that fundamentally the underlying determinant of the rate of innovation appears to be something that is generally outside the model (i.e. is exogenous). More specifically, the growth rate of labor (or of population) largely depends on demographic influences (birth, death, migration) rather than, say, technology policies. In this sense, the model has been referred to as a semi-endogenous growth model because the long run determinant of g is exogenous and because technology policies only affect g temporarily, having long run effects only on the level of R&D but not the growth rate of R&D. But some semantic issues do exist here. If we define L more specifically as the stock of *skilled* labor, technology and education policies could influence the steady state rate of accumulation of human capital and thus of the growth rate of L over the long run.

3.2. Product Proliferation

Under this approach, scale effects are pre-empted because if larger economies imply more people, a larger market implies more diverse preferences and hence a greater demand for a variety of products or solutions to similar problems. Consequently, aggregate research and development is spread out among more sectors or firms over time as the economy expands in size. The rate of innovation would thus not increase with scale. This product proliferation approach is the approach used in Young (1998), Dinopoulos and Thompson (1998), and Howitt (1999), among others.

To implement this approach, the above canonical models are modified by introducing both horizontal and vertical differentiation. For example, one could replace (1b) with

$$D_t = \exp\left(\int_0^{n(t)} \ln q^J(j)\, c(j)\, dj\right) \qquad (1b)'$$

where the measure of variety (n) can increase over time, and/or replace (2b) with

$$Y_t = \left(\int_0^{n(t)} x(j)^{\beta}\, dj\right) L_Y^{\,1-\beta} \qquad (2b)'$$

In the absence of horizontal innovation, an increase in the size of the economy creates opportunities for economic rents. These rents are typically exhausted through vertical innovation. That is, an innovator responds to the opportunities for economic rent by developing a better quality good which then destroys the economic rents associated with previous innovations. Hence the economic rents associated with an increase in scale are eventually exhausted but in the process a higher rate of innovation results. Thus the solution to removing scale effects is in introducing horizontal innovation. The latter is another means by which to exhaust the economic rents associated with a larger scale, namely through increased product varieties and entry of firms (that supply them). The expansion of horizontal goods and sectors in response to larger scale uses up R&D resources and helps maintain the overall growth rate of the quality of goods.

However, some restrictive assumptions are imposed under this second approach, perhaps the most stringent of which is that there exist knowledge spillovers associated with vertical innovation but not with horizontal innovation. Such spillovers would allow researchers to freely build on knowledge created by others – but only on knowledge created through quality improvements. This specification or assumption is not based on any empirical findings or theoretical microfoundations, but is used as an aid to solving the models. The role of horizontal innovation is to put a check on scale effects – to prevent changes in scale from affecting the long run growth rate of vertical innovation. Increased horizontal innovation essentially absorbs resources and helps dilute R&D resources by having them spread out over a larger number of research projects.

With product proliferation, the long run growth rate is as follows:

$$g = \frac{\dot{n}}{n} = \frac{1}{A}\left(\frac{L_R}{N}\right) \qquad (34)'$$

where N is an index of product variety. Note the resemblance to the growth rate under the R&D difficulty approach (where N replaces Z). In contrast to the canonical models, the growth rate of innovation is influenced not by the (absolute) level of R&D workers but by the ratio of R&D workers to N, where N can be related to the size of the economy, whether gross domestic output, population, or labor force. The greater N is, the greater the variety of needs. As long as the index N grows at the same rate as L_R, g will not change in response to variations in the absolute level of L_R.

Typically, but not always, N is parameterized as a function of labor; for example, $N = L^{\sigma}$. Some models assume $\sigma = 1$, in which case the innovation rate depends not on the level of R&D labor but the share of R&D labor in the total labor force. However, Jones (2005) makes a good point about the assumption that $\sigma = 1$ – that it has a "knife-edge" property. In particular if $\sigma < 1$, scale would matter. The growth in number of sectors would be less than the growth in number of workers, leading to a rise in the size per sector. On the other hand if $\sigma > 1$, the size per sector would decline leading to a negative scale effect.

Implementing the product proliferation models empirically could also run into some challenges, particularly since data on R&D or patents is typically not classified as vertical or horizontal. The empirical researcher would have to assess each research outcome, rather subjectively, as to whether the innovation is in variety or quality. Many innovations combine ideas from different innovations, as well as improve existing ideas or create new products. This makes it hard to classify these blended inventions as a vertical innovation or horizontal (e.g. cell-phone with camera, or i-Pod phone).

In comparison to the R&D difficulty approach, the product proliferation modifications to the canonical models do not result in a semi-endogenous growth outcome where the long run rate of innovation depends on exogenous determinants. Rather, the equilibrium innovation rate shown in (34)' indicates the potential role for technology and trade policies to affect g if they affect the relative level of R&D labor or the ratio of R&D resources to N (whatever N may be, whether aggregate labor or output, or other).

4. Policy I: R&D Subsidies

I now turn to some policy applications. In this section, the focus is on the impact of R&D subsidies on long run innovation. The conventional view is that because subsidies reduce the cost of R&D, innovation should respond positively. However, second-round factors do need to be taken into account. For example, subsidies may be financed via distortionary taxation which could offset the positive effects.

The innovation growth literature has identified other issues to consider. It turns out that the efficacy of R&D subsidies on innovation depends on the role of scale effects – hence the intimate connection between scale and subsidies in this literature. But, there is no clear consensus in the literature as to what the long run impacts of subsidies are on innovation growth. As background, Jones (1995) removes scale effects by adopting the R&D difficulty approach. As was seen in the previous section, the result is that the equilibrium innovation growth rate depends on exogenous factors (e.g. demographics). Hence technology policies such as R&D subsidies would not affect g (the growth rate). At most, such policies affect the level of innovation but not the growth rate of innovation.

In Young (1998), technology policies such as R&D subsidies also do not affect the growth rate of quality innovations. Like increases in scale, an increase in subsidies generates economic rents, which are then dissipated via expansions in horizontal innovation, so as to leave the long run rate of growth of quality intact. However, because greater product variety results, the increase in subsidies does increase welfare (or consumer utility) even though the underlying growth rate of the economy is unaffected. Howitt (1999) shows that the neutral effect of subsidies on innovation depends on Young's (1998) assumption that horizontal and vertical innovation have identical technologies. Howitt (1999) allows the returns to vertical innovation to decline less rapidly than the returns to horizontal innovation, and finds that subsidies can affect the long run rate of innovation.

As a further example of the diversity of theoretical results on subsidies and innovation, Segerstrom (1998) finds a nonlinear relationship between the optimal subsidy and the size of innovation quality. The larger the size of the quality jump, the larger is the markup, and the greater the incentive the innovator should have to innovate. Thus the greater that price exceeds marginal cost, the less useful a role that a subsidy has. Indeed if the quality jump is sufficiently large, there may be excessive incentives to innovate in which case an R&D tax is optimal. Thus the optimal subsidy has an inverted-U relationship with innovation size: rising with size and then falling after the size reaches a critical point, and possibly becoming negative in value for sizes that are very large.

This result is in contrast to the result in case 3, where we saw that subsidies would enhance innovation if innovation jumps were of intermediate size. Recall that it is in the case of intermediate sized jumps where the private market under-invests in quality improvements. Otherwise for small jumps and for large jumps, a tax would be optimal. The presence of scale effects in case 3 can explain the difference in results. For example, if we allow for R&D difficulty (see the variable Z in (34)), then for small sizes of innovation, there is less impact on R&D difficulty. Innovations are initially more frequent and short-lived (for small λ). Hence there is likely to be too little R&D early on, a situation that could justify subsidies. It is when innovation sizes get larger that R&D difficulty is greater so that the R&D sector uses more and more resources for research projects, calling for a tax when too many resources are allocated to R&D.

Li (2001) criticizes the finding in Segerstrom because it suggests that incremental innovations should get more favorable treatment from policy than major, drastic innovations. Li (2001) develops a model where both drastic and non-drastic innovations may occur, and where knowledge spillovers can occur within and between industries. The consequence is that subsidies may be optimal for augmenting the stock of drastic innovations. The intuition is that the business stealing effect which causes R&D investment to be excessive no longer varies with innovation size for drastic innovations, and is not a factor to consider in setting the optimal subsidy policy.

Thus far, we have not distinguished between subsidies to variety innovation and subsidies to quality innovation. For instance, Dinopoulos and Thompson (1998) find that subsidies to horizontal innovation slow economic growth while subsidies to vertical innovation increase economic growth. This is not surprising in that knowledge spillovers exist only with quality improvements. Only vertical innovation, as a by-product, facilitates further innovation by others.

But in many cases, the policymaker may not target subsidies for one type of innovation or another, or may not be able to observe which type of R&D is geared towards quality improvements and which type towards variety growth. Thus policymakers would tend to provide "general" subsidies for innovation. Nonetheless, a general subsidy could have differential effects on vertical and horizontal innovation. Indeed, Segerstrom (2000) illustrates that the relative economic efficacy of subsidies depends on two conditions: (i) the extent to which general subsidies favor one type of R&D (say, to develop new goods) over another (say, to improve the quality of existing goods); and (ii) the extent to which one type of R&D is more profitable and conducive to furthering technological change. To illustrate this, consider the product proliferation approach (2b)' above, where intermediate inputs undergo both quality improvements and expansions in variety. The aggregate market clearing condition is:

$$Y = \left(\int_0^n q(j)\, x(j)^{\beta} dj \right) L_{\mathrm{Y}}^{1-\beta} = C + I_{\mathrm{H}} + I_{\mathrm{V}}$$

where I_{H} denotes investment in horizontal innovation and I_{V} in vertical innovation. Let g_{q} denote the growth rate of the quality index and g_n the growth rate of variety:

$$g_{\mathrm{q}} = g_{\mathrm{q}}(v^{\delta_{\mathrm{V}}}), \quad 0 < \delta_{\mathrm{V}} < 1$$
$$g_n = g_n(h^{\delta_{\mathrm{H}}}), \quad 0 < \delta_{\mathrm{H}} < 1$$

where $v = \dfrac{I_{\mathrm{V}}}{Y}$ and $h = \dfrac{I_{\mathrm{H}}}{Y}$ are the R&D investment rates. The growth rates of quality and variety are positive diminishing functions of the R&D investment rates. If $\delta_{\mathrm{V}} > \delta_{\mathrm{H}}$, the diminishing returns to vertical R&D are smaller than those to horizontal R&D.

Let the index of R&D difficulty for vertical innovation be given by $Z = q^{\omega}$, where q is the latest quality level. Next, as a measure of economic growth, we use the growth rate of the real wage (or marginal productivity of labor):

$$g = g_q + (1-\beta)g_n$$

In other words, real wage growth is driven by a combination of quality and variety growth, where variety growth has a factor of influence of $(1-\beta)$, which is the output elasticity of labor. If $\omega > \frac{1}{1-\beta}$, technological change is faster under horizontal R&D than under vertical R&D. The intuition is that if ω is large, conducting vertical R&D becomes more increasingly difficult, so that horizontal R&D is the more profitable activity and stronger engine of growth. With these two important conditions, four cases are possible depending on $\delta_V <$ (or $>$) δ_H and on $\omega < (\text{or} >) \frac{1}{1-\beta}$. Box 1 shows the predictions associated with each case.

Box 1. Long Run Effects of General Subsidy:

	Subsidy favors Vertical R&D ($\delta_V > \delta_H$)	Subsidy favors Horizontal R&D ($\delta_V < \delta_H$)
Vertical R&D is More Profitable ($\omega < \frac{1}{1-\beta}$)	Rate of Innovation Increases	Rate of Innovation Decreases
Horizontal R&D is More Profitable ($\omega > \frac{1}{1-\beta}$)	Rate of Innovation Decreases	Rate of Innovation Increases

Due to differences in the speed of diminishing returns a general subsidy could favor one type of R&D over another, and as long as the subsidy favors that type of R&D which has the greater potential to speed up technological growth, the overall rate of innovation and economic growth can increase.

To summarize, the theoretical effects of R&D subsidies on private sector innovation are varied. From a policy standpoint, the positive and normative effects of subsidies are uncertain, and little guidance on how to conduct policy under such circumstances is offered. From an empirical standpoint, the theories here provide conditional predictions. It may be much more useful in empirical work to assess the conditions or environment under which R&D subsidy policies operate; for example, whether vertical or horizontal R&D experiences the greater diminishing returns or the extent to which business stealing effects vary with innovation size.

5. Policy II: Patent Protection

Thus far, it has been assumed the patent rights of innovators are perfectly enforced. In practice, innovators face risks of imitation and infringement, not just displacement by superior innovations. Intellectual property rights issues have generated much controversy. The heart of the debate is whether stronger patent rights stimulate innovation. On the surface, stronger patent protection (against imitation and infringement) seems to provide technology developers with greater incentives to develop new technologies or to conduct R&D. However, if patent protection is too strong so as to shield technology owners from competitive pressures or from technology rivals, the technology owner may have less incentive to innovate and create new products which would only destroy the economic rents from his existing products. On the other hand, even if patent rights are not too strong as to preclude rivalry from other innovators, the other innovators may face a higher cost of conducting R&D if they are charged more to utilize existing inventions, compared to a situation where technological inputs are in the public domain. In other words, licensing fees and royalties add to the cost of conducting R&D.

Thus patent systems are complex. At the very least, it is important to recognize the tradeoffs. By granting an inventor a patent, the system gives the patent holder the exclusive rights to produce and sell the invention and to determine whom to give licenses, if any, for the use of the invention. The patent holder in essence acquires market power (though not necessarily monopoly power if the patent holder is not the only firm in the industry). This creates *static inefficiencies* (as prices exceed marginal cost) and the diffusion of the good is less widespread than it if were competitively supplied. However, without a patent right, the inventor may never have developed the invention. Without the ability to charge a price above marginal cost, the inventor would not be able to recoup the R&D cost of development. Since inventive knowledge is a public good (non-rival and non-excludable), rivals could produce and sell the good without themselves undertaking the R&D cost of development. With competition, the price of the good would be driven down to marginal cost. The inventor would thus earn no economic profits to recoup the R&D investments. Anticipating this, inventors may avoid investing in innovation. Hence patent systems help “correct” a market failure. In this sense, patent rights with temporary duration create dynamic efficiencies. Hence, policymakers need to strike a balance between the dynamic efficiencies and the static inefficiencies.

In this section application of the dynamic models in this chapter to study the effects of patent rights on innovation is illustrated. Two models are examined, one based on horizontal innovation and the other on vertical innovation. Both models show that the patent system is most conductive to innovation and welfare if patent rights are neither too weak nor too strong.

5.1. Horizontal Innovation

The first model is an extension of case 2 (variety innovation in intermediate inputs) and is based on Kwan and Lai (2003). Stronger patent protection is shown to involve a tradeoff between short run consumption and long run consumption growth. The intuition is that in the short run, stronger protection reduces imitation risk and increases the incentive of innovators to do more variety R&D. The expansion in R&D requires resources that would otherwise, in the short run, have been available for current consumption. Furthermore, producers face an increase in the cost of production due to the reduced availability of cheaper imitated inputs. In the long run, the increased expansion in varieties and stock of knowledge increase consumption opportunities.

To illustrate this idea, assume a variety of intermediate inputs. Some fraction of them are imitated and supplied competitively. The non-imitated inputs are each supplied by a monopolist.

We modify the production function as follows:

$$Y = \left(\int_0^n x(j)^\beta \, dj \right) L_Y^{\ 1-\beta} = \left(\int_0^{n_c} x_c(j)^\beta \, dj + \int_{n_c}^{n} x_m(j)^\beta \, dj \right) L_Y^{\ 1-\beta} \qquad (35)$$

where n_c is the measure of imitated inputs and $(n - n_c)$ the measure of non-imitated inputs.

On the expenditure side:

$$Y = C + F\dot{n} + \int_0^{n_c} x_c(j)dj + \int_{n_c}^{n} x_m(j)dj \tag{36}$$

where F is the fixed product development cost of innovating a new variety, which is paid to the R&D sector (recall Eq. (20)). The inputs, x, are produced with labor. The manufacturer's demand for inputs is:

$$p_i(j) = \frac{\partial Y}{\partial x_i(j)} = \beta\, x_i(j)^{\beta-1} L_Y^{\;1-\beta}, \qquad i = \mathrm{c}, \mathrm{m} \tag{37}$$

The intermediate goods producer faces the demand curve given by (37). Its profits are:

$$\pi_i(j) = (p_i(j) - w)\, x_i(j), \qquad i = \mathrm{c}, \mathrm{m}$$

where one unit of labor is used to produce one unit of output. Using the first-order condition for profit maximization, we can derive the optimal production of x, price of x, and the maximum profits. The producers of non-imitated inputs $j \in (n_c, n]$ each have market power over their own variety input:

$$x_m(j) = \left(\frac{\beta^2}{w}\right)^{\frac{1}{1-\beta}} L_Y \tag{38}$$

$$p_m(j) = \left(\frac{w}{\beta}\right) \tag{39}$$

$$\pi_m(j) = \left(\frac{1-\beta}{\beta}\right)\left(\frac{\beta^2}{w}\right)^{\frac{1}{1-\beta}} L_Y \tag{40}$$

The producers of imitated inputs $j \in [0, n_c]$ behave competitively:

$$x_c(j) = \left(\frac{\beta}{w}\right)^{\frac{1}{1-\beta}} L_Y \tag{38'}$$

$$p_c(j) = w \tag{39'}$$

$$\pi_c(j) = 0 \tag{40'}$$

Firms, as before, profit from having exclusive rights to commercialize an innovation but face risks of imitation. Assume

$$\dot{n}_{\mathrm{c}} = \frac{1}{\theta}(n - n_{\mathrm{c}}) \qquad (41)$$

This is the rate at which proprietary innovations are copied. Assume the strength of patent protection is given by the parameter θ.

The market value of being an exclusive supplier, as before, is the present discounted value of profits $V = \frac{\pi_{\mathrm{m}}}{r_{\mathrm{m}}}$, where $r_{\mathrm{m}} = r + \frac{1}{\theta}$ is the effective discount rate (i.e. the interest rate plus the risk of imitation). The risk of imitation is inversely related to the index of patent strength, θ.

The free-entry condition (8b) then is:

$$V = \frac{\pi_{\mathrm{m}}}{r + \frac{1}{\theta}} = F$$

I continue to assume log-utility, so that the Euler equation is:

$$g = \frac{\dot{C}}{C} = r - \rho = r_{\mathrm{m}} - \frac{1}{\theta} - \rho$$

Define $E = \frac{C}{nF}$ and $\iota = \frac{n_{\mathrm{c}}}{n}$ where E is consumption per resources spent on knowledge and ι the share of imitated goods, respectively. Both E and ι are the key state variables of the model. Using Eqs. (41), (36), (37), (38), and (38)' – and normalizing $w = 1$ – yields the following system of dynamic equations:

$$\frac{\dot{\iota}}{\iota} = \frac{1}{\theta}\left(\frac{1}{\iota} - 1\right) + \Lambda_1 \iota + E + \Lambda_2 \qquad (42a)$$

$$\frac{\dot{E}}{E} = E + \Lambda_1 \iota + g + \Lambda_2 \qquad (42b)$$

where E is the "jump" variable and ι the predetermined variable, and where $\Lambda_1 = -r_{\mathrm{m}}(\beta^{\frac{-1}{1-\beta}} - \frac{1}{\beta} - 1) < 0$ and $\Lambda_2 = -r_{\mathrm{m}}\left(\frac{1+\beta}{\beta}\right) < 0$.

In steady-state,

$$\iota^* = \frac{1}{1 + \theta g}$$

$E^* = \Lambda_1 \iota^* - g - \Lambda_2$

The main results are:

$$\frac{\partial \iota^*}{\partial \theta} = \frac{-g}{(1+\theta g)^2} < 0 \tag{43a}$$

$$\frac{\partial E^*}{\partial \theta} = -\Lambda_1 \frac{\partial \iota^*}{\partial \theta} - \frac{1}{\theta^2} < 0 \tag{43b}$$

Stronger patent protection reduces the share of imitated goods in the long run and results in a higher steady-state ratio of knowledge to consumption (that is, more varieties per consumption spending).

However, to differentiate the short-run impact and long run consequences of stronger patent protection, note how the path of consumption spending changes over time. From solving the Euler equation, we can derive:

$$C_t = C_0\, e^{g(\theta)t} = F\, n_0 E_0(\theta) e^{g(\theta)t} \tag{44}$$

Thus, a change in the strength of patent rights leads to:

$$\frac{\partial \ln C_t}{\partial \theta} = \frac{1}{E_0(\theta)} \frac{\partial E_0(\theta)}{\partial \theta} + \frac{\partial g(\theta)}{\partial \theta} t \tag{45}$$

Figure 1 graphically illustrates the result in (45): at the instant that patent protection is strengthened the level of consumption (in natural logs) falls but grows at a higher rate thereafter.

Figure 1: Time path of (the natural log of) consumption in response to a permanent increase in patent protection at time t_0

To determine the optimal θ, we maximize the lifetime utility function (1) subject to (44). The necessary condition is:

$$\frac{\partial U}{\partial \theta} = \underbrace{\frac{1}{\rho E_0} \frac{\partial E_0(\theta)}{\partial \theta}}_{"-"} + \underbrace{\frac{g'(\theta)}{\rho^2}}_{"+"} = 0 \tag{46}$$

The first part is the marginal cost of increased protection (in terms of the decline in short run consumption) and the second part the marginal benefit (from a higher rate of growth in consumption over the long run). Thus Eq. (46) captures the dynamic tradeoff associated with stronger patent protection.

5.2. Vertical Innovation

Another perspective on the relationship between innovation and patent protection can be seen in a model of vertical R&D, as developed by O'Donoghue and Zweimuller (2004). This model utilizes Eqs.

(1), (1b), (3b), (4), (8a), and (8b) and yields an inverted-U relationship between the rate of innovation and patent strength as measured by *patent breadth*, of which there are two types. Lagging breadth determines the range of inferior products that infringe, and leading breadth determines the range of superior products that infringe. The focus here is on leading breadth.

Suppose that the latest patent holder in a particular product line has a quality innovation that is λ^* times better than the previous generation or version of the good. Let κ denote the patent breadth policy whereby new innovations of quality in the range

$$\lambda \in [\lambda^*, \kappa\lambda^*] \qquad \kappa > 1$$

are infringing. The new innovator must then seek and negotiate a license and pay royalties accordingly. Thus every innovator receives royalties and licensing fees from his technology as well as pays royalties and licensing fees in order to use other innovators' technologies. Thus let B denote an index of bargaining strength. The greater B is, the more rents the patent holder collects than he pays out.

Thus a key change in specification concerns the firm's value (Eq. (8a)):

$$V = \frac{\pi(\kappa)\, B(\kappa)}{\phi + \rho}$$

Increasing leading breadth, κ, has two opposing effects on R&D incentives. On the one hand, the patent holder enjoys larger markups and thus larger profits π. This creates greater incentives to do quality R&D. On the other hand, the patent holder has a weak bargaining position vis-à-vis incumbent patent holders early in the life of a patent. The new patent holder's innovation is likely to step on existing patent rights, requiring various licensing fees to be paid It is only later in the life of a patent, when the rights of existing patent holders expire, that the patent holder is in a stronger bargaining position with respect to the next generation of patent holders. In this sense, the payoffs to a patent are backloaded, arriving later in time. Thus a larger κ increases the bargaining power of incumbent patent holders and weakens that of new patent holders, hence lowering B. Overall R&D increases only if the payoffs to a patent are not too backloaded; that is, if the fall in B does not overwhelm the rise in π. Hence, R&D may have an inverted-U relationship with breadth (rising initially and falling after κ reaches a certain point).

5.3. Further Results

Other studies concur that a nonlinear (inverted-U) relationship exists between patent strength and innovation: that stronger patent protection stimulates innovative activity up to a point, beyond which stronger patent rights reduce innovative activity. For example, Horowitz and Lai (1996) model vertical innovation and find that a longer duration of patent protection increases the size of quality jumps but makes them less frequent. The optimal patent length should therefore balance the desire for higher quality with the desire for more timely introductions of innovations. Aghion et al. (2001) argue that while imitation reduces the profitability of innovation, some imitation is better than none. The reason is that the presence of imitators creates greater "neck and neck" competition between the incumbents and rivals, thus giving leaders incentives to escape "product market competition" – in other words, to avoid being displaced.

The right balance for patent protection strength is also likely to vary with the economic environment. Grossman and Lai (2004) and Angeles (2005) point out that the optimal level of patent protection depends on the level of economic development and innovative capacity. For example, the optimal strength of patent rights in developing economies should be lower than that in developed economies.

As in the case of R&D subsidies, the influence of patent policies could also depend on whether scale effects exist. The issue is whether variations in patent policy can affect the level and/or the growth rate of long term innovation. The focus in this section has been on the incentives for innovation and the costs and benefits of patent rights, rather than on the levels versus rates debate. Whether scale effects exist or not, the tradeoffs concerning IPRs are qualitatively similar. Indeed O'Donoghue and Zweimuller (2004) re-analyze their model after purging it of scale effects and find qualitatively similar conclusions about the role of patent breadth on innovation incentives.

6. Open Innovation

As discussed in the previous section, patent rights have the potential to stimulate R&D but they do create economic costs. Goods are not priced at marginal cost. The cost of performing innovation is higher, as technology users are required to pay royalties and licensing fees for patented technologies. Due to these disadvantages of a system of proprietary innovation, an *open innovation* movement has occurred within certain innovation communities.

Open innovation refers to a system that relies on free and open development. Innovators do not assert patent rights to inventions. They freely share and disseminate their knowledge, discoveries, or innovations. Examples of open innovation include the open source movement in software where source code is freely shared and where users can modify programs and the open biotechnology movement where innovators also freely share and adapt agricultural, pharmaceutical, and biotechnological research output. A survey on the open source movement is provided in Isaac and Park (2004), and a survey on open-biotech is provided in Isaac and Park (2005).

The literature has largely focused on explaining the motivation for participation; for example, why is open innovation rational? What is the ulterior motive of open innovators? Lerner and Tirole (2002) discuss reputation considerations and how participation in open source projects can enhance a programmer's long term career prospects. Chesbrough (2003) argues that open innovation may at times be a more profitable strategy than closed innovation, and provides a number of case studies where firms like Xerox, IBM, Intel, Proctor and Gamble, Merck, and so forth found that giving open access to their technologies helped increase the value of their technologies and created opportunities for further innovation and commercialization.

Economic history also provides examples of the profitability of open innovation. Allen (1983) discusses the technological development of the blast furnace and Nuvolari (2003) the development of the pumping engine. Both were developed during the 19th century in the U.K. by an open community of innovators. How did the free dissemination of inventions ultimately benefit the innovators? Often the value of complementary assets of the innovators increased with the adoption and diffusion of the new technology. For example, the new technologies helped raise the value of the ore mines and ore deposits. Another reason innovators benefited from open innovation is that the engineers at the time faced much technological uncertainty. They had no idea how modifications to the design of furnaces or to pumping engines would turn out. One way in which engineers could collectively infer whether a solution worked or not is to allow the widespread copying of designs. The more designs that are copied, the more observations engineers would have to assess their work. In other words, engineers would have a greater sample size with which to test their "theories" or solutions.

Von Hippel (2005) characterizes the traditional closed-innovation approach as rather manufacturer-centric. Users merely express their needs and the manufacturers identify and fulfill them. In an open innovation community, users actively participate in innovative activity. They are in a better position to identify their needs, and by actively participating, they are better able to develop solutions that are customized to their needs. The users then reveal their knowledge or innovations so as to encourage the manufacturing and widespread adoption of their innovations. Following this idea, Harhoff et al. (2003) develop a theoretical model where free-revealing increases a user's chance that his or her innovation would be widely adopted, holding other factors constant.

In contrast to studies on motivation and participation in open innovation, there exists little theoretical work to date examining the positive impacts of open innovation, for example on the economy-wide rate of innovation. A recent exception is Saint-Paul (2003), which extends one of the dynamic canonical models described in this chapter to include open innovation activities. One especially useful feature of this model is that it incorporates both proprietary and open innovation. Theoretical models that focus on only one type of innovation may miss out on the joint interactions; for example, open source communities may generate much innovation but if they "crowd out" proprietary innovation, the overall (economy-wide) change in innovation is ambiguous.

Thus the Saint-Paul (2003) model is a useful starting point for a theoretical analysis of open innovation. In this model, free (nonproprietary) goods coexist with proprietary goods. Saint-Paul (2003) uses the term "free good", but the good in question is not free in the sense that its sales price is zero. Indeed, consumers do have to pay a price for it, namely the marginal cost of production. What Saint-Paul (2003) means by free is that the technology for creating it is available for free (in the public domain). No licensing or royalty fees are associated with it. The inventor of the technology does not expect nor receive compensation. However, once a design is created, it costs to manufacture each unit. A supplier then simply charges the marginal cost of manufacturing the good.

This is a model of horizontal innovation in final goods. Let us begin with the consumer whose static utility function is a modification of Eq. (1a):

$$D=\left[\int_0^n c(j)^\alpha\, dj\right]^{\frac{1}{\alpha}} = \left[\int_0^{n_F} c_F(j)^\alpha\, dj + \int_{n_F}^{n} c_P(j)^\alpha\, dj\right]^{\frac{1}{\alpha}} \qquad \text{(1a)'}$$

where c_F denotes nonproprietary goods and c_P proprietary (patented) goods.

As before, the model features symmetry among nonproprietary goods and among proprietary goods, so that (1a)' can be rewritten as:

$$D= \left[n_F c_F{}^\alpha + (n-n_F) c_P{}^\alpha\right]^{\frac{1}{\alpha}} \qquad (47)$$

Labor is required to manufacture both nonproprietary and proprietary goods. Each unit of output requires one unit of labor, hence the amount of workers required in manufacturing is $L_Y = n_F c_F + n_P c_P$, where $n_P = n - n_F$. Labor is also required for R&D activities. However, it is assumed that labor devoted to the creation of free goods (denoted by L_F) does not detract from labor required for manufacturing or for proprietary R&D. Labor devoted to inventing nonproprietary goods is purely philanthropic (e.g. spent during weekends or evenings) and thus does not pose any problem for resource allocation in the private sector. This assumption is made primarily to analyze the other aspects of open innovation (i.e. how the latter affects profitability). If we allowed philanthropic labor to compete with other kinds of labor, then open innovation could have direct crowding out effects. We discuss this impact later. Thus if L is the endowment of labor for private sector use:

$$L = L_Y + L_R$$

where L_R, as before, is the labor employed in R&D.

The consumer solves both a static and dynamic problem. Here, the model is such that aggregate consumption is constant over time (and hence $r = \rho$). In the static problem, the consumer maximizes (47) above subject to the budget constraint

$$Income = w n_F c_F + \frac{w}{\alpha} n_P c_P$$

where the right hand side represents expenditure. The price of the nonproprietary good is the wage, w. This good is available at marginal cost, whereas a markup (w/α) is charged for the proprietary good. The solutions to the maximization problem are:

$$c_F = \frac{L_Y}{\xi} \qquad (48a)$$

$$c_P = \alpha^{\frac{1}{1-\alpha}} \frac{L_Y}{\xi} \qquad (48b)$$

where $\xi = n_F + \alpha^{\frac{1}{1-\alpha}} n_P$.

The instantaneous profit to the (typical) proprietary firm is:

$$\pi = (p - w) c_P = (\frac{1}{\alpha} - 1) w \alpha^{\frac{1}{1-\alpha}} \frac{L_Y}{\xi} \qquad (49)$$

Now, let us characterize the innovators. The philanthropic innovator's knowledge production function is a variant on (3a):

$$\dot{n}_F = \frac{L_F}{A} n \qquad \text{(Non-proprietary Varieties)}$$

$$\dot{n}_{\mathrm{P}} = \frac{L_{\mathrm{R}}}{A} n = \frac{(L - L_Y)}{A} n \qquad \text{(Proprietary Varieties)}$$

where L_{F} is the labor devoted to philanthropic research (and again, it is assumed that there is no time conflict between the two activities: engaging in one type of research does not take time and resources away from the other). The main difference between these knowledge production functions and (3a) earlier is that here proprietary and non-proprietary innovators can both build off one another's research. Knowledge spillovers within and between sectors exist; hence, the term "n" appears on the right-hand side rather than just the knowledge or varieties that each group has created.

The evolution of proprietary and non-proprietary varieties is related. For example, under balanced growth:

$$g = \frac{\dot{n}_{\mathrm{F}}}{n_{\mathrm{F}}} = \frac{\dot{n}_{\mathrm{P}}}{n_{\mathrm{P}}}$$

Moreover, from the knowledge production functions, both sectors have equal average productivities:

$$\frac{\dot{n}_{\mathrm{F}}}{L_{\mathrm{F}}} = \frac{\dot{n}_{\mathrm{P}}}{L_{\mathrm{P}}} = \frac{n}{A}$$

Combining these two factors (balanced growth and equal productivities) yields a relationship between manufacturing labor and non-proprietary labor (or an indirect influence of L_{F} on L_Y).

$$L_Y = L + L_{\mathrm{F}} - gA \qquad (50)$$

This equation shows that a larger amount of philanthropic R&D labor leads to the creation of more varieties and increased employment opportunities in manufacturing. We can now substitute Eq. (50) into Eq. (49) to complete the expression for the profits of the proprietary firm. As before, there is a no-arbitrage condition:

$$\frac{\dot{V} + \pi}{V} = r$$

where V is the market value of the proprietary firm (equal to the present discounted value of profits). The free entry condition $V = \dfrac{wA}{n}$ also needs to be satisfied. Substituting V above into the no-arbitrage condition along with the expression for profits (using (49) and (50)) yields:

$$\left(r - \frac{g(1-2\alpha)}{\alpha}\right)\frac{A}{n} = \frac{\alpha^{\frac{1}{1-\alpha}}(L + L_{\mathrm{F}} - Ag)}{\xi} \qquad (51)$$

This gives us an implicit nonlinear relationship between the growth rate of innovation (g) and nonproprietary R&D labor (L_{F}) – that is, $g = g(L_{\mathrm{F}})$. Our interest is determining the nature of this relationship.

Totally differentiating (51) and evaluating it at $L_{\mathrm{F}} = 0$ and $r = 0$ (i.e. around small variations in nonproprietary labor and in the opportunity costs of innovation) shows that:

$$\frac{\partial g}{\partial L_{\mathrm{F}}} > (\text{or} <)\ 0 \quad \text{according to whether} \frac{1}{\alpha} + \frac{1}{\alpha}^{\frac{-1}{1-\alpha}} > 2 \ .$$

The pricing markup (above marginal cost) is $\frac{1}{\alpha}$. In order for non-proprietary R&D to stimulate long run innovation, this markup needs to be about 1.727 or higher. In other words, only if the markup is 72.7% above wages would nonproprietary R&D stimulate the overall rate of innovation. Anything less would mean that, while the growth rate of nonproprietary varieties increases, the growth rate of proprietary varieties decreases more, so that on balance a lower economy-wide rate of innovation results.

Intuitively, increased nonproprietary R&D has two opposing effects. On the one hand, it generates knowledge spillovers that increase both proprietary and nonproprietary varieties. It also increases the employment of workers in manufacturing, incomes of consumers, and the spending power of consumers. The increased incomes translate into increased demand for both proprietary and nonproprietary goods. Hence, the profits of proprietary firms should increase and thereby give incentives for proprietary R&D, hence contributing to higher equilibrium rates of innovation. However, the expansion of nonproprietary goods, which are cheaper, tends to lower the market share of proprietary firms. This reduces their profits and market value, reducing incentives for proprietary R&D. The latter (negative) effect dominates so long as the markup is low. Markups are smaller if nonproprietary goods are closer substitutes for proprietary goods, or if proprietary firms have weaker market power. Otherwise, markups that are sufficiently high could compensate for the decline in the market share of proprietary goods. Thus, even though nonproprietary inventions incur no direct resource cost (i.e. philanthropic labor effort is not compensated monetarily), indirectly they could either stimulate or reduce aggregate incentives to conduct R&D, depending on the size of the markup.

This section is closed with some criticisms of the model. First, it depicts competition between open and proprietary developers at the product level. However, a lot of what goes on in open source activities is sharing at the research level. The goal often is to maintain the shared access to enabling, platform technologies and preserve fundamental research tools in the public domain. In other words, it would be useful to model innovation occurring not just in final goods or even in intermediate inputs, but in research inputs (like software programming code or expressed sequence tags in biotechnology).

Moreover, in the model above, users or consumers of nonproprietary goods pay for the goods. However, in most open innovation settings, goods or inputs are actually available for free. The nominal price is zero. Consequently, there is scope for externalities or spillovers in consumption or production. This potential is not modeled here. It would be useful to examine open source activities as a source of knowledge externalities.

Lastly, there is a tendency to think of patent systems and open innovation as opposites or substitutes. To some degree, that is true. However, as Isaac and Park (2004) argue, that it not always or necessarily the case. For example, open innovation tends to produce customized solutions while proprietary innovators tend to develop solutions for the mass market. Open innovation has especially helped serve the needs of the professional, sophisticated users, whereas proprietary producers have served the end-user market. Thus open and proprietary innovation may each have its own comparative advantages. Moreover, both open innovation and property rights may be complementary. Indeed it is not true that open innovators completely forgo the use of intellectual property rights. The community as a whole often seeks brand name protection, copyrights, and/or patents. The difference is that the patented inventions are available for free (without royalties). The free use of patented invention does, however, come with licensing restrictions. For example, the user agrees not to assert proprietary rights over innovations that build on the community's patents. In the end, the open innovation communities use intellectual property rights to help ensure that the communities' innovations remain free and open.

7. Concluding Remarks

In this chapter, four dynamic models of innovation have been surveyed. Innovation is reflected in the variety and quality of either final goods or intermediate inputs. These models were then used to address the role of scale (market size) effects and of technology policy, such as R&D subsidies, patent protection, and open innovation (as an alternative or complement to proprietary innovation).

On scale effects, we pointed out the empirical controversy as to whether larger economies actually have higher rates of innovation and grow faster. Scale effects are tempered or eliminated if R&D becomes increasingly difficult as the level of innovative activity increases or if innovation produces a proliferation of industries or sectors among which R&D resources get spread more thinly. The absence or presence of scale effects is critical to determining whether subsidies to R&D can affect the long run rate of innovation. Scale effects tend to diminish the impact of R&D subsidies on growth, but the ultimate impact of subsidies appears to depend on whether they are biased towards horizontal innovation or vertical innovation, and whether vertical or horizontal R&D is the more critical engine of economic growth.

On institutional matters like patent systems, we argued that stronger patent rights create both costs and benefits for innovation, and that ultimately it is a matter of finding the appropriate balance. In a dynamic context, the optimal level of patent protection should weigh the short run cost of reduced consumption (as resources are allocated towards innovative activity) against the long run benefit of a higher growth rate of consumption (as the economy accumulates a greater stock of innovations). To mitigate the costs of patent systems, some innovation communities engage in open innovation, where knowledge and inventions are shared freely. Both good economic incentives and efficiency considerations are associated with this mode of innovation. However, using a model where both proprietary and open innovation occur alongside each other, we pointed out the possibility that expanding open innovation may crowd out for-profit innovation, directly or indirectly, so that the overall change in economy-wide innovation is ambiguous. While this is a possibility, we also argued that there are good reasons to think of open innovation and the patent system, say, as complements, particularly where there are niche markets and where open innovators focus more on infrastructural technologies which all researchers could benefit from accessing.

Due to space considerations, a number of important subjects on innovation have not been discussed in this chapter and which the interested reader could pursue. For example, the focus in this chapter has been exclusively on private sector innovation. It should be noted that the public sector also accounts for a significant share of research and development. For a theoretical model of horizontal R&D and government funded research, see Park (1998) and for a discussion of private and public research collaboration, see Scotchmer (2004, chapter 8). Secondly, models in a closed economy setting have been considered. A vibrant literature also exists on the impact of trade on innovation and technology transfer in a North-South context. The North refers to the industrialized world and the South to the developing world. For a survey of North-South trade and innovation, see Chui et al. (2002). Third, the models in this chapter implicitly assume that innovations, once created, are diffused and adopted immediately or easily. However, in practice, innovations may sit on the shelf for some time. It would be useful to study in more detail the processes by technology adoption and diffusion occur. For a survey of this literature, see Hall and Khan (2003).

Glossary

Economic Rent: Payment to an input that exceeds the minimum needed for that input to be supplied.
Externalities (or Spillovers): A positive externality occurs if an economic activity generates benefits even to those who do not share the cost of the activity. A negative externality occurs if an economic activity generates harm by agents who do not pay for the damage.
Final Good: Good produced for final consumption.
Horizontal Innovation: An innovation resulting in a new variety of good.
Intermediate Input: Good used as an input into the production of final goods.
Marginal Cost: Cost of producing an additional unit of output.
Markup: The factor by which the price of a good exceeds its marginal cost.
Open Innovation: A system under which innovation is non-proprietary and occurs under free and open development conditions.
Proprietary Good: A good that is produced for-profit and exclusively under the control of a producer with a property right.
Scale Effect: A positive effect of a larger economy on the equilibrium rate of innovation.
Social Optimality: A condition under which social welfare or well-being is maximized.
User Innovation: Innovation stimulated by end users. Such users develop innovations for their own use because existing goods do not fulfill their specific needs.
Vertical Innovation: An innovation resulting in an improved quality of good.

Glossary of Symbols

U	Utility
D	Index of consumption
C	Total consumption expenditure
c	Consumption per good
E	Consumption per knowledge capital
Y	Total output
n	Measure of variety of goods (or inputs)
q	Index of quality
λ	Size of quality jumps
p	Price of a good (or input)
w	Wage per worker
r	Nominal interest rate
ρ	Time preference rate
π	Firm profits
V	Firm value
L	Total resources (labor)
L_Y	Labor employed in production
L_R	Labor employed in research and development (R&D)

L_F	Philanthropic labor
K	Capital
I	Investment
x	Intermediate input
Z	Index of R&D difficulty
ω	Parameter measuring the rate at which R&D becomes more complex
N	Index of Product Proliferation
F	Fixed costs of innovation
g	Growth rate of varieties
ϕ	Probability of research success
A	Parameter measuring the inverse productivity of R&D workers
α	Parameter characterizing tastes for variety
β	Parameter measuring the elasticity of capital inputs
δ	Parameter measuring returns to R&D
θ	Index of patent rights
ι	Share of imitated varieties
κ	Breadth of patent rights
B	Index of patent holder's bargaining strength in licensing negotiations

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